

Here are some observations about the writing of mathematics that I hope will be useful as you work on the writing assignment for this course.

**Goals and Audience.** As with any written piece of mathematical exposition, you must be clear about your audience and your specific goals in mind. Be sure you have a clear sense of what these are before you start writing.

**The Process.** It is important to bear in mind that writing is a *process*, just like proving a theorem. None of us forthrightly organizes a clear, error-free exposition the first time. The first time you sit down to write, just as no one produces a complete, well-structured proof the first time, the thing about a problem is not good. The first draft has been thoroughly rewritten at least once or twice before it reaches the reader with a few sections undergoing perhaps three to five major revisions. To some extent, this thought makes the prospect of writing seem daunting or even overwhelming, but it need not be the idea that much of what you write will eventually be replaced or discarded. Just sit down and write, knowing that anything that doesn't measure up can later be deleted. You might wonder how you can throw away the first several pages you write; this is not waste time since the trial-and-error process helps you immensely in clarifying what you really want to say.

When you begin writing a draft, the introduction may not be the best place to start, since the structure of the paper may not become completely clear until later in the process. Start somewhere in the middle with whichever part of the paper is clearest in your mind as soon as you have a section or more in relatively coherent form. Sit back and read it out loud from time to time. If you see if it makes complete sense, then rewrite.

When you have something you think is close to acceptable, give it to someone else to read and comment on. Then rewrite again.

After you think the paper is finished, go through it with a net to check and sharpen your definitions, statements of theorems, and proofs. Clarify your logic and your intuitive descriptions. Be sure your setting, punctuation, and grammar are also useful, correct, and need less words, terminology, and symbols.

Note that rewriting usually means much more than simply correcting errors. It means looking critically at what you've written both locally and globally, figuring out what works and what doesn't, and doing whatever is necessary to make the whole thing work effectively.

**Conventions.** Though you might not believe it after reading some of the mathematical writing that has made it into print, mathematical writing should follow the same conventions of grammar, usage, punctuation, and setting as any other writing. This means in particular that you must write complete sentences organized into paragraphs. Here, mathematical terms have technical meanings that are different from their usage in ordinary English. You should still be careful to observe the usual rules regarding articles of speech and subject-verb agreement, though you will run across a few too many mathematicians who

**recision** In mathematical writing more than another, a recision is of a greater amount.

correct use of the sentence, let  $f$  be the function whose value at a particular number is equal to the square of that number, and let  $f$  be the function whose value at a particular number is equal to the number itself. It is far less clear than, let  $f$  be the function defined by  $f(x) = x^2$  or  $f(x) = x$ . On the other hand, the excessive use of symbols can lead to writing that is just as obscure.

Here are some guidelines for using mathematical symbols in your writing.

- Use the minimum number of names/symbols to the minimum necessary for clarity. Ideally, each symbol should refer to an object whose role in the paper or in a particular section of the paper is important enough that you want the reader to remember it by name. This is especially true in statements of theorems. Which of the following statements is clearer?

**Theorem** *If  $G$  is any Lie group, there exists a Lie group  $\tilde{G}$  that is the universal covering group of  $G$ .*

**Theorem** *' Every Lie group has a universal covering group.*

Of course, even if you use the second version in the proof of the theorem, you will probably want to introduce symbols such as  $G$  and  $\tilde{G}$  to refer to the given group and its universal cover.

- Using symbols and short simple formulas should be included right in your paragraphs, but a formula that is large or especially important should be centered on a line by itself. This is called a display formula.
- Verbal mathematical symbols or formulas, whether included in the text or as a separate must have a definite grammatical function in a sentence, usually as a noun or a clause. Consider the following sentence:

If  $x > 0$ , we see that  $x^2 > x$  must be greater than

Here the formula  $x > 0$  is a clause whose verb is  $>$ , while  $x^2 > x$  functions as a noun.

- If a separate formula ends a sentence, it must be followed by a period.
- The best way to ensure that your formulas function grammatically correctly is to read each sentence aloud. When you do so, bear in mind that many symbols can be read in several different ways. For example, the symbol  $\int$  can be read as "equals", "integral", "be equal to", or "is equal to", depending on context.

- Symbols representing mathematical relations, i.e.,  $>$  or  $\in$  or operators, i.e.,  $-$  or  $\cap$  should be used only to connect other mathematical symbols, not words. For example, do *not* write

Let  $\mathbf{v}$  be a vector of length  $<$  that is  $\in T_x M$ .

Instead, this sentence could be rewritten as follows:

Let  $\mathbf{v} \in T_x M$  be a vector such that  $|\mathbf{v}| <$

- Fractions and fractional expressions included in the text should be written with a slash as in  $x/2$ . If a fraction is so large or complicated that it needs to be written using a horizontal bar, it should be written as a separate line. The only exceptions are small numerical fractions such as  $\frac{1}{2}$  which can be included in text as long as the

- The symbols of symbolic logic such as  $\exists$   $\forall$   $\wedge$   $\vee$   $\neg$  and  $\Rightarrow$  should never be used in formal mathematical writing unless you are writing about symbolic logic and then they appear in logical formulas. Otherwise write out the words instead.

**Citations** Whenever you write a mathematics paper you must list in your bibliography every published source from which you obtain ideas, mathematical results, proofs, facts, or specific language. Whenever you write something that you obtained from such a source you must refer specifically to the source in the text. If you use a large amount of material from one source as you might do in an expository paper, it is permissible to write something like "all of the results in this section are from [1]" but it is not enough just to list a paper or book in your bibliography. Here are some examples of situations that require citations:

The bibliography. The conventions for bibliographic references in mathematical writing are somewhat different from those in other fields. Some bibliographic information is shown below. The first entry is for a book; the second is for an unpublished preprint in an online database.

- [1] J. R. Dieudonné, P. R. Halmos, M. M. Schiffer, and N. E. Steenrod, *How to write mathematics*, American Mathematical Society, Providence, 1981. Look especially at the essay by Halmos, which is a classic.
- [2] L. Gillman, *Writing Mathematics Well*, Mathematical Association of America, 1987.
- [3] D. E. Knuth, T. Larrabee, and P. M. Roberts, *Mathematical Writing*, Mathematical Association of America, Washington, 1989. Look especially at pages 1-8.