PTRL2020

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7cbj Ybcfg

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: cf`W ffYbhigh XYbhgžU``Ybei]f]Yg`UbX`Ugg]gHUbW fY`Uh]b[`hc`Ybfc`a YbhžWUgg`fY[]ghfUh]cbždfc[fYgg]cb WYV<u>Vg</u>`UbX`ch\Yf`UXa]b]ghfUh]j Y`a UhYfgžd`YUgY`gYY`<u>H\Y`Bi WYi g.`Gh XYbh<i V</u>"

K YV/ '=a dcfHJbh'@]b_g.' <u>GW&cc``cZA]bYfU`g`UbX`9bYf[mF Ygci fWfg`</u> <u>H\Y`Bi WYi g`Ghi XYbh<i V</u> <u>A ccX`Y</u> <u>I BGK '<UbXVcc</u> <u>I BGK 'H]a YHVYY</u> <u>Ghi XYbhK Y``VY]b[</u> <u>I f[YbhA YbHJ``<YU`h`/ `Gi ddcfh</u> <u>9ei]HVY`@YUfb]b[`GYfj]Wfg</u>

7cifgY[·]8YHJ]`g

I b]hg`cZ'7 fYX]h'*

Gi a a UfmcZh\Y'7ci fgY

H\Y'Z bXUa YbHJ'g'cZXck b\c'Y'[Ycd\mg]WU``c[[]b[`fk Y``!`c[[]b[E'UbX``c[`]bHYfdfYHJfjcb'UfY'Wcj YfYX']b h\]g'Wci fgY'''K Y``!`c[[]b[`]g'i gYX']b'gYj YfU``]bXi ghf]Yg'gi W\'Ug'DYhfc`Yi a ž'A]b]b[ž'7]j]``UbX`=bZfUghfi Wi fY UbX'9b[]bYYf]b['; Yc`c[mhc'cVHJ]b'h\Y'Zcfa Uh]cb'dfcdYfh]Yg'Ufci bX'h\Y'VcfY\c`Y'UWW fUhY`mk]h\`\][\ gdUh]U``fYgc`i h]cb'''K Y``!`c[[]b[`XUhU`WUb'dfcj]XY'bi a Yfci g'cddcfhi b]h]Yg'hc'YI hfUWhZcfa Uh]cb W\UfUWhYf]gh]Wg'Zca 'a YW\Ub]WU`'hc'Zck'dfcdYfh]Yg'UbX'UfY']a dcfhUbhZfca 'YUfmghU[Y'cZYI d`cfUh]cb'h`UhY ghU[Y'cZUVUbXcba YbhUbX'Wcgi fY''

7ci fgY⁻5]a g

 ⊕ iħ]g Wei fgYžghi XYbhg UfY]bhfcXi WYX hc fYgYfj c]f fcW_UbX Zi]X dfcdYfh]Yg UbX YUfb ħ Y Z bXUa YbhU cZ k Y c[]b[UbX c[]bhYfdfYhUh]cb "H\Y]bhY[fUh]cb cZZi]X!fcW_dfcdYfh]Yg iħ fci [\ WefY UbU mg]g U cb[k]ħ k Y ``! c[]bhYfdfYhUh]cb Zefa g ħ Y Zei bXUh]cb cZfYgYfj c]f Yj U`i Uh]cb 5g dUfhcZh Y Wei fgYžghi XYbhg UfY U`gc]bhfcXi WYX hc fYU k Y ``! c[XUhU Zefei U`]mrWebhfc žUbU mg]g UbX]bhYfdfYhUh]cb"

7cifgY`@YUfb]b[`CiHWcaYg

- %"I bXYfgHUbX]b[`h\Y`dYhfcd\mg]WU``dfcdYfh]Yg`cZh\Y`fYgYfj c]fg`UbX`h\Y`k Umg`h\YmUfY`a YUgi fYX`]b fci h]bY`UbX`gdYW]U``WcfY`UbU`mg]g`k cf_Zck "
- &"CVHU]b]b[`_bck`YX[Y`cZd\mg]WU``df]bW]d`Yg`cZkY```c[[]b[`UbX`H\Y`hcc`g`igYX`hc`aYUgifY dYhfcd\mg]WU``dfcdYfh]Yg"
- ' ": i bXUa YbHJ`gʻcZei UbHHUHj YʻUbX`ei U`HUHj Yʻ]bHYfdfYHUHjcb`cZk Y``!`c[`XUHU"
- ("G]a d`Y`]bhYfdfYfUh]cb`cZU`gYhcZfYU``kY``!`c[`XUfU`i g]b[`[U]bYX`_bck`YX[Y`k]h\]b`[fci dg"

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5 'gYhcZfYU``XUHU']g'dfcj]XYX'UbX']bhYfdYfUh]cb'cZh\Y'XUHU']g'YI dYWhYX'''

8i Y XUHY. K YY_(8YUX`]bY'Zcf'UVgc`i hY'ZU]`. "9I hYbg]cb'g\ci `X'VY'fYei YghYX'k]h\'dfcdYf'fYUgcb]b["

5ggYgga Ybh(@c[]bhYfdfYhUh]cb

H\Y`ZJbU``YIUa`]g`ZcWgYX`cb``c[`]bhYfdYhUh]cb`U`h\ci[\`U```dUfhg`cZh\Y`WcifgY`UfY`WcjYfYX`]b`h\Y`YIUa"

5ggYgga YbhWf]hYf]U

:]bU``9IUa 'k]``VY`fi b`cb`U```gYWgcbg`cZh\Y`Wci fgY`]b`dUff]W``Uf`h\Y``c[`]bhYfdfYHUgcb"

8YUX`]bY'Zcf'UVgc`i hY'ZU]`. "5bma]gUXj Ybh fY'g\ci 'X'VY'Zc``ck YX'k]h\'U'GdYWU`'7 cbg]XYfUh]cb"

5ggYgga Ybh' 🗄]bU` 9l Ua

8YUX`]bY'Zcf'UVgc`i hY'ZU]`. "5bmZJ]`i fY'hc'UhhYbX''UV'gYgg]cbg'k]``'VY'fYj]Yk YX'Vm8f'7\Yb

@UVcfUhcfmigYgg]cb'a i gh'VY'UhYbXYX

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@UV k cf_jg WebXi WhYX cb (k YY_g UbX UfY U`cWUhYX he dY hfed \gm]WU`dfedYfhmia YUgi fYa Ybhg cZfeW_g]b h\Y``UV"`5`fYdcfhcZh\Y``UV`k cf_`]g`YI dYWhYX`Uhih\Y``Ughk YY_"

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4

5HYbXUbW/FYei]fYaYbhg

fi]g'YI dYWYX'N Uhighi XYbhk]```UhYbX'Uh`YUghi, \$i `cZhi hcf]U`g'UbX``YWi fYg"'Bcfa U``mž'N YfY']g'bc'a U_Y! i d'k cf_'Zcf'dccf'UhYbXUbWY'UbX'ghi XYbha][\hiZU]``H\Y'Wci fgY"'=Zmci `\Uj Y'a]gUXj Ybhi fY'cf']`!\YU'N ž d`YUgY'WcbhUWhimci f'Wci fgY'WccfX]bUhcf'Ug'gccb'Ug'dcgg]V'Y"'H\Y'UhYbXUbWY'fYei]fYa Ybh]g'bcha YUbhhc VY'di b]hjj Y"'=fi]g']bWi XYX'VYWUi gY'dUfh]W]dUh]cb']g'Ub']a dcffUbhdUfhcZUWX]Yj]b['h\Y'Wci fgY'ci hWca Yg"

7 ci fgY GWX YXi `Y

J]Yk WUgg ha YHUY Y

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8 UHY	Hmd Y	7 cbhYbh
K YY_`(.`% `>i bY`!`&' >i bY	5ggYgga Ybh	@c[`]bhYfdfYhUh]cb
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6UhYa UbžFž@c[<code>`EiU`]hm7cbhfc`ž%, ("</code>

6UhYa UbžF`A žCdYb'<c`Y`@c[`5bU`mg]g`UbX': cfa Uh]cb'9j U`i Uh]cbž'≢bhYfbUh]cbU``<i a Ub`FYgci fWYg 8Yj Y`cda Ybh7cfdcfUh]cbž6cghcbž%,)"

6f]h]g\`DYhfc`Yia`7c"'@hXžCif`=bXighfmDYhfc`Yiaž'>Uffc`X'/`GcbgžBcfk]W\ž%++"

7`Uf_žBž'9`Ya Ybhg`cZDYhfc`Yi a `F YgYfj c]fgž'GD9'GYf]Ygž'% *\$"

7 cfY@UVž: i bXUa YbHJ`g`cZ7 cfY`5bU`mg]gž'% +' "

8YgVfUbXYgžFž9bWMWcdYX]UcZKY``@c[[]b[ž%,)"

8YkUbž'>ž'9ggYbhJU`g`cZAcXYfb`CdYb`<c`Y`@c[[]b[ž'%,'"

8fYggYf 5hUgž K Y```@c[[]b[`UbX` +bhYfdfYhUh]cb HYW kb]ei Ygž %, &"

 $8 fYggYf^{5}hUg\check{z}^{e}c[\dot{t}bhYfdfYhUh]cb^{7} \ Ufhg\check{z}^{*}$,)"

<Y`UbXYfž'8ž': i bXUa YbłU`g`cZ: cfa Uł]cb'9j U`i Uł]cbž'%, ' "

<]`WX]Yž'8ž'5dd`]YX'CdYb'<c`Y'@c['CdYfUh]cbgž'%,&"

@mbWXž'9ž: cfa Uh]cb'9j U`i Uh]cbž'% *&"

D]fgcbžGž; Yc`c[]WK Y```@c[`5bU`mg]g"'GWX`i a VYf[Yfž@c[`+bhYfdfYhUh]cb`Df]bW]d`Yg#5dd`]WUh]cbgž%, -

GWX`i a VYf[Yfž@c[`+bHYfdfYHUH]cb'7\Ufhgž'%-)"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

H\Y'GW\cc``\Ug'XYjY`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'giVa]hhjb['U'WcifgY'Ugg][baYbh'

K Y`YbWci fU[Y`mci `hc`fYHJ]b`U`WcdmicZYj YfmiUgg][ba Ybhigi Va]hHYX`Zcf`UggYgga YbhZcf`mci f`ck b`fYWcfX Y]h\Yf`]b`\UfXWcdmicf`Y`YWfcb]WZcfa "

5```UggYgga Ybhg`a i gh\Uj Y`Ub`UggYgga YbhWcj Yf`g\YYhUhUV&YX"

7cifgYWcad`Yh]cb

7 ci fgY'Wca d`Yh]cb'fYei]fYg'gi Va]gg]cb'cZU```UggYgga Ybh']hYa g"': U]`i fY'hc'gi Va]h'U```UggYgga Ybh']hYa g a Um'fYgi `h']b'h\Y'Uk UfX'cZUb'I bgUh]gZUWcfm: U]`i fY'fI : ½[fUXY'Zcf'h\Y'7 ci fgY'i b`Ygg'gdYW]U` Wcbg]XYfUh]cb'\Ug'VYYb'gi Va]hHYX'UbX'Uddfcj YX"

@UhY Gi Va]gg]cb cZUb 5gg][ba Ybh

: i ```a Uf_g'Zcf'Ub'UggYgga YbhUfY'cb`mdcgg]V`Y'k \Yb'Ub'UggYgga Ybh]g'fYWf]j YX'Vmh\Y'Xi Y'XUhY'''K cf_ gi Va]hhYX``UhY'k]h\ci hUb'Uddfcj YX'YI hYbg]cb'Vmh\Y'Wci fgY'WccfX]bUhcf'cf'XY`Y[UhYX'Ui h\cf]m]g'gi V'YVh hc'U``UhY'dYbU`hmcZZj Y'dYfWfbhf) i EcZh\Y'a UI]a i a 'a Uf_'dcgg]V`Y'Zcf'h\UhUggYgga Ybh']hYa "'H\Y``UhY dYbU`hm`]g'Udd`]YX'dYf'WU`YbXUf'XUmf]bWi X]b['k YY_YbXg'UbX'di V`]W\c`]XUmgE'h\UhIh\Y'UggYgga Ybh']g cj YfXi Y"'H\YfY']g'bc'dfc!fUhU'cZh\Y``UhY'dYbU`hmZcf'gi Va`]gg]cbg'a UXY'dUfhk Umh\fci [\`U`XUm''H\]g']g Zcf'U```UggYgga Ybhg'k \YfY'U'dYbU`hmUdd`]Yg"

K cf_`gi Va]hhYX`UZhYf`Z]jY`XUmg`f%&\$`\ci fgŁ`k]``bch`VY`UVWYdhYX`UbX`U`a Uf_`cZnYfc`k]``VY`Uk UfXYX`Zcf h\Uh`UggYgga Ybh`]hYa "

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hma Umbch`VY`Uddfcdf]UhY"`H\YgY`k]```VY`]bX]WUhYX`]b`h\Y`Wci fgY ci h`]bYž`UbX`gi W\`UggYgga Ybhg`k]```fYWY]j Y`U`a Uf_`cZnYfc`]Zbch`Wca d`YhYX`Vm'h\Y`gdYW]ZJYX`XUhY" 9I Ua d`Yg`]bWi XY.

- K YY_`mcb`]bY`hYghg`cf``UVcfUhcfmk cf_`k cfh\`U`ga U```dfcdcfh]cb`cZh\Y`gi VYWha Uf_žcf
- C b`]bY'ei]nnYg'k \YfY'Ubgk Yfg'UfY'fY`YUgYX'hc'gh XYbhg'cb'Wca d`Yh]cbž'cf'DfcZygg]cbU` UggYgga YbhHUg_gž'k \YfY'h\Y']bhYbh]cb']g'hc'WfYUhY'Ub'Ui h\Ybh]WUggYgga YbhH\Uh\Ug'Ub'UVgc`i hY gi Va]gg]cb'XUhYž'cf'DUgg# U]`'UggYgga YbhHUg_g"

K Y'i bXYfgHUbX'H\Uh'Uh'h]a Yg'mci 'a Umbch'VY'UV'Y'hc'gi Va]h'Ub'Ugg][ba Ybh'cb'h]a Yž'UbX'H\Y'GW\cc``k]`` UWWca a cXUHY'UbmZU]f'UbX'fYUgcbUV'Y'YI hYbg]cb"'K Y'k ci `X'fYWca a YbX'mci 'fYj]Yk 'H\Y'I BGK 'GdYW]U` 7 cbg]XYfUh]cb'[i]XY`]bYg'-gYW]cb'VY'ck "

GdYWJU``7cbg]XYfUh]cb

Gh XYbhGi ddcfh

H\Y'I b]j Yfg]mUbX'h\Y': UW `mdfcj]XY'U'k]XY'fUb[Y'cZgi ddcfhgYfj]Wg'Zcf'gh XYbhgž']bWi X]b[.

- @VfUfmihfU]b]b['UbX'gi ddcfhigYfj]Wfg'! '<u>k k k "`]VfUfm'i bgk "YXi "Ui</u>
- 5WUXYa]WG_]``g`Gi ddcfh! <u>`\hdg.#k k k "gh XYbh'i bgk "YXi "Ui #g_]``g</u>
- DgmWkc`c[miUbX`K Y``bYgg`! <u>k k k "Wci bgY``]b["i bgk "YXi "Ui</u>

9ei]HUV`Y`@YUfb]b[`GYfj]WYg`U]a g`hc`dfcj]XY`U```gh`XYbhg`k]h\`U`ZYY`UbX`WcbZ]XYbh]U``gYfj]WY`h\Uh dfcj]XYg`dfUWh]WU``gi ddcfhhc`Ybgi fY`h\Uhmci f`\YU`h\`WcbX]h]cb`XcYgbfhiUXj YfgY`mUZYWhmci f`ghi X]Yg"` <u>\hhdg.##qh`XYbh'i bgk "YXi "Ui #Y`g</u>

5WUXYa]W<cbYghmUbX D`U[]Uf]ga

Mci f``YWi fYf`UbX'h\Y`I b]j Yfg]mik]``YI dYWimci f`gi Va]hYX`Ugg][ba Ybhg`UfY`hfi `mmci f`ck b`k cf_"`I BGK \Ug`j YfmWYUf`[i]XY`]bYg`cb`k \Uhd`U[]Uf]ga `]g`UbX`\ck 'hc`Uj c]X`]h'`D`U[]Uf]ga `]g`i g]b[`h\Y`k cfXg`cf]XYUg`cZch\Yfg`UbX`dfYgYbh]b[`h\Ya `Ug`mci f`ck b"`D`U[]Uf]ga `]g`U`hmdY`cZ]bhY``YWi U``h\YZt``±i\Wb`hU_Y a UbmZcfa gž`Zca `XY`]VYfUhY`W\YUh]b[`hc`UWW]XYbhU``mWcdm]b[`Zca `U`gci f\W`k]h\ci hUW_bck`YX[Ya Ybh' H\Y`I b]j Yfg]hm\Ug`UXcdhYX`Ub`YXi WUh]j Y`UddfcUV\\`hc`d`U[]Uf]ga `UbX`\Ug`XYj Y`cdYX`U`fUb[Y`cZ fYgci f\W`g`hc`gi ddcfhghi XYbhg"`5```h\Y`XYhU]`g`cb`d`U[]Uf]ga ž`]bWi X]b[`gca Y`i gYZ ``fYgci f\W`gž`\Ub`VY Zci bX`Uh<u>k k k "ghi XYbh'i bgk "YXi "Ui #d`U[]Uf]ga</u>."

5```A 9F 9`ghi XYbhg`UfY`fYei]fYX`hc`Wca d`YhY`U`ghi XYbhXYWUfUh]cb`Zcf`UWUXYa]W]bhY[f]hrik \]W\`]g ci h]bYX`]b`h\Y`Ugg][ba YbhWcj Yf`g\YYhg"`6mig][b]b[`h\]g`XYWUfUh]cbžmci `U[fYY`h\Uhmci f`k cf_`]g`mci f ck b`cf][]bU``k cf_"

Zmci 'bYYX'gca Y'UXX]h]cbU``gi ddcfhik]h\ mci f'UWUXYa]Wg_]``gžd`YUgY'WcbHUWhH\Y'5WUXYa]WG_]``g Gi ddcfhicfij]Yk 'gca Y'cZh\Y'fYgci fWYg'cb'h\Y]f'k YVg]hY. <u>\hhdg.#k k k "ghi XYbh'i bgk "YXi "Ui #g</u>_]``g"'H\Y 5WUXYa]WG_]``g`HYUa 'WUb'dfcj]XY'fYgci fWYgžgi ddcfhiUbX'Ugg]gHUbWY'hc'\Y`d'mci `]a dfcj Y'mci f UWUXYa]Wg_]``g"Gca Y'ghi XYbhg`i gY'h\Y'7YbhfY'gYfj]WYg'VYWUi gY'h\YmUfY'Z]bX]b['h\Y]f'Ugg][ba Ybhg`U

•

5W_bck `YX[Ya YbhcZ7ci bhfm

 $\label{eq:constraint} \begin{array}{l} K_{1}^{T}(UW_{bck}^{T}X_{l}^{T}) & K_{1}^{T}(UX_{l}) \\ F(UX_{l}) & F(UX_{l}) \\ F(UX_{l}) & F(UX_{$