

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] h m	@cWUh]cb	D\cbY
; i m5``]bgcb	["U``]bgcb4 i bgk "YXi "Ui	@YWhifYihja Yg	Н9Н6	
: i feUb'@Y! <i ggu]b<="" td=""><td></td><td>@YWhifY"hja Yg</td><td>Н9Н6</td><td></td></i>		@YWhifY"hja Yg	Н9Н6	

GWk.cc``7**Gh/kh/a/c**ið **27 fa huhll/khó zðu 2c ffa** d**ulþjucb**

GWX&c``cZA]bYfU`g`UbX`9bYf[mF Ygci fWYg C`X`A U]b`6i]`X]b[ž`@Yj Y``%z`%) - `fP %) Ł I BGK `GM8B9MBGK `&\$) & 51 GHF 5@=5

: cf'W ffYbhgh XYbhgž'U```Ybei]f]Yg'UbX'Ugg]ghUbW fY`Uh]b['hc'Ybfc`a Ybhž'WUgg'fY[]ghfUh]cbž'dfc[fYgg]cb WXYW_g'UbX'chXYf'UXa]b]ghfUh]j Y'a UhYfgž'd`YUgY'gYY'<u>H\Y'Bi WYi g.'Gh XYbh<i V</u>

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5ggYgga Ybh& A]X!hYfa 'hYgh

 $\label{thm:linear} $H\setminus g^hYghg^hY'gh^Y'gh^XYbhgfii bXYfghUbX]b[\ ^cZbYhWUg\ ^Zck\ ^UbU`mg]g^UbX^YWcbca\]W]bX]WUhcfg$

5ggYgga Ybh' . :]bU`Yl Ua

5 'ZjbU``YI Ua 'Wcj Yf]b['hcd]Wg 'Zfca 'h\Y``UhhYf`\U`ZcZh\Y'Wci fgY"

5 h Y b X U b W F Y e i] f Y a Y b h g

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YHUV'Y

H]a YtUV`Y

8 UhY	Hmd Y	7 cbhYbh
K YY_'%'%': YVfi Ufm! %+': YVfi Ufm	@YWhi fY	BYh7Ug\`:`ck
K YY_'&.'&\$': YVfi Ufm! &(': YVfi Ufm	@YWhi fY	BYh7Ug\:`ck
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K YY_") . "% "A UfVX "! "%+ A UfVX	@YWhi fY	7 UgY '91 Ua d'Yg
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A UfVX		
K YY_', .'' '5df]`!'+'5df]`	@YWn fY	F]g_'5bU`mg]g
K_YY_''%\$'5df]`'!'%(5df]`	@YWhi fY	F]g_`5bU`mg]g
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Fygci fWyg

DfYgWf]VYX F Ygci fWYg

7 ci fgY 'bchYg 'UbX 'YI YfW]gYg ']b 'A ccX 'Y

FYWca a YbXYX FYgci fWYg

7 ci fgY 'bchYg 'UbX 'YI YfW]gYg ']b 'A ccX 'Y

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

: YYXVUW_'Xi f]b['ei]nnYg

@UVcfUhcfmK cf_g\cd \DZcfa Uh]cb

Bch'Udd`]WUV`Y

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

H\Y'GW\cc``\Ug'XYjY`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'giVa]hjb['U'WcifgY'Ugg][baYbh'

K Y YbWci fU[Y'mci'hc'fYhJ]b'U'WcdmcZYj YfmUgg][ba Ybhgi Va]hYX'Zcf'UggYgga YbhZcf'mci f'ck b'fYWcfX Y]hYf']b'YUfXWcdmcf'Y'YWfcb]WZcfa "

5 "UggYgga Ybhy a i gh\Uj Y Ub UggYgga YbhWcj Yf g\YYhUHUW\YX"

7ci fgY Wca d`Yhjcb

7 ci fgY'Wca d`Yhjcb'fYei]fYg'gi Va]gg]cb'cZU``'UggYgga Ybh']hYa g": U]`i fY'hc'gi Va]h'U``'UggYgga Ybh']hYa g a UmfYgi `h']b'h\Y'Uk UfX'cZUb'l bgUhjgZJWcfm: U]`i fY'fl : Ł'[fUXY'Zcf'h\Y'7 ci fgY'i b`Ygg'gdYW]U` Wcbg]XYfUhjcb'\Ug'VYYb'gi Va]h\X'UbX'Uddfcj YX"

@UhY Gi Va]gg]cb cZUb 5gg][ba Ybh

: i ```a Uf_g'Zcf'Ub'UggYgga Ybh'UfY'cb`mdcgg]V`Y'k \Yb'Ub'UggYgga Ybh']g'fYW']j YX'Vmh\Y'Xi Y'XUhY''K cf_gi Va]h\YX'`UhY'k]h\ci hUb'Uddfcj YX'YI hYbg]cb'Vmh\Y'Wci fgY'WccfX]bUhcf'cf'XY`Y[UhYX'Ui h\cf]m']g'gi V'YWh hc'U'`UhY'dYbU'micZ]j Y'dYfWYbhf) i E'cZh\Y'a Ul]a i a 'a Uf_'dcgg]V`Y'Zcf'h\Uh'UggYgga Ybh']hYa "'H\Y'`UhY'dYbU'mi]g'Udd`]YX'dYf'WU'YbXUf'XUmf]bWi X]b['k YY_YbXg'UbX'di V`]W\c`]XUmgE'h\Uhh\Y'UggYgga Ybh']g cj YfXi Y'''H\YfY']g'bc'dfc!fUh'JcZh\Y'`UhY'dYbU'miZcf'gi Va]gg]cbg'a UXY'dUfhk Umh\fci [\'U'XUm''H\]g']g Zcf'U``'UggYgga Ybhg'k \YfY'U'dYbU'miUdd`]Yg"

K cf_'gi Va]hhYX'UZhYf'Zjj Y'XUmg'fl%&\$`\ci fgŁ'k]```bch'VY'UWWYdhYX'UbX'U'a Uf_'cZnYfc'k]```VY'Uk UfXYX'Zcf h\Uh'UggYgga Ybh']hYa "

: cf`gca Y`UggYgga Ybh`]hYa gž`U``UhY`dYbU`hm`a Umbch`VY`Uddfcdf]UhY''`H\YgY`k]```VY`]bX]\WJhYX`]b`h\Y`\\ci fgYci h`]bYž`UbX`gi \W`UggYgga Ybhg`k]```fY\\]j Y`U`a Uf_`cZnYfc`]Zbch\\can d`YhYX`\Vmh\Y`gdY\\]ZJYX`XUhY'' 9I Ua d`Yq`]b\Wi XY.

- K YY_`mcb`]bY 'hYghg'cf'`UVcfUhcfmk cf_'k cfh\ 'U'ga U``'dfcdcfh]cb'cZh\Y'gi VYWha Uf_žcf
- C b`]bY'ei]nnYg'k \YfY'Ubgk Yfg'UfY'fY'YUgYX'hc'ghi XYbhg'cb'Wca d`Yh]cbz'cf'DfcZYgg]cbU`
 UggYgga YbhiNug_gz'k \YfY'h\Y']bhYbh]cb']g'hc'WfYUhY'Ub'Ui h\Ybh]WUggYgga YbhiNuh\Ug'Ub'UVgc`i hY
 gi Va]gg]cb'XUhYz'cf'DUgg# U]`'UggYgga YbhiNug_g"

K Y'i bXYfghJbX'h\UhiUhih]a Yg'mci 'a UmbchVY'UV'Y'hc'gi Va]h'Ub'Ugg][ba Ybhicb'h]a Yž'UbX'h\Y'GW\cc``k]`` UWWca a cXUhY'UbmZJ]f'UbX'fYUgcbUV'Y'YI hYbg]cb"'K Y'k ci `X'fYWca a YbX'mci 'fYj]Yk 'h\Y'I BGK 'GdYWJU` 7 cbg]XYfUh]cb'[i]XY`]bYg'—'gYY'gYWfJcb'VY`ck "

GdYWJU`7cbg]XYfUhJcb

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9ei]HUV`Y'@YUfb]b['GYfj]WYg'U]a g'hc'dfcj]XY'U```gh XYbhg'k]h\'U'ZYYY'UbX'WcbZ]XYbh]U``gYfj]WY'h\Uh dfcj]XYg'dfUWf]WU``gi ddcfhihc'Ybgi fY'h\Uh'mci f'\YU'h\'WcbX]h]cb'XcYgbfhiUXjYfgY`mUZZYWhimci f'ghi X]Yg''`\hdg.##gh XYbh'i bgk "YXi "Ui #Y`g

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

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5W_bck `YX[Ya YbhcZ7ci bhfm

5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

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?bck `YX[Y'UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yžh\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhU`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY	
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