# **PTRL6027**

## 7Ug]b[ '8Yg][ b'UbX'7Ya Ybh]b[



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VFI

## 7 ci fgY<sup>-</sup>Cj Yfj ]Yk

### GHJZZ7cbHJWhi8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
N\]I] <sup>.</sup> 7\Yb	<u>n\] ]W4 i bgk "YXi "Ui</u>	CZ]₩ſ∖cifg	C Z]WF &%%ž @Yj Y``&ž H9H6	Ž*%&'-',))%&

#### GWXcc``7cbHJWhi=bZcfaUh]cb

GW&cc``cZA]bYfU`g`UbX`9bYf[mFYgcifWYg C`X`AU]b`6i]`X]b[ž`@YjY``%z`%) - `fP`%) Ł IBGK `GMBB9MBGK`&\$)&5IGHF5@=5

<u>9b[]bYYf]b[ 'Gh XYbhGYfj ]Wg</u>'

9. a YfY"hYUW\]b[4 i bgk "YXi "Ui

K. <u>k k k "Yb[]bYYf]b["i bgk "YXi "Ui #a</u>]bYfU`g!YbYf[mfYgci fWg

## 7ci fgY<sup>·</sup>8YHJ]`g

#### l b]hg cZ7fYX]h\*\*

#### Gi a a UfmcZhXY7ci fgY

H∖Y`W¢i fgY`W¢j Yfg.

7 Ug]b[`8Yg][b`!`5D=dfcdYft]Yg`cZWUg]b[`UbX`WUg]b[`Wcid`]b[g/dYfZcfaUbWY`dfcdYft]Yg`cZWUg]b[`ibXYf` `cUX`WcbX]t]cbg/df]bW]d`Yg`cZWUg]b[`XYg][b`Zcf`jYft]WU`žXYj`]UhYX`UbX`\cf]ncbtU``kY``g/dfYdUfUt]cb`cZ WUg]b[`dfc[fUag`Zcf`X]ZYfYbtikY```hmdYg/cdt]a]gUt]cb`cZWUg]b[`dfc[fUa"

7 Ya Ybhjb[ `-7 Ya Ybha Ubi ZUWi fYžWca dcg]hjcb'UbX'gHUbXUfX]nUhjcb/a YUgi f]b[ 'UbX'WcbHc``]b[ 'WYa Ybh dfcdYfhjYg/WYa YbhUXX]hj Yg/WYa Ybhg`i ffmXYg][ b/7 Ya Ybhjb[ 'WU'W `Uhjcbg'! Df]a UfmWYa Ybhjb[ ž'd`i [ VU`UbWjb[ ž'UbX'gei YYnY'WYa Ybhjb[ /f\Yc`c[ mUbX'hmdYg'cZZck /a YW\Ub]ga 'cZa i X'fYa cj U``VmWYa Ybh/ WYa Ybhjb[ 'Yei ]da Ybh'd`Ubb]b[ žWcbXi Whjb[ 'UbX'a cb]hcf]b[ 'df]a UfmUbX'gYWcbXUfmWYa Ybhjb[ 'cVg/dcgh 'cV'Wcbg]XYfUhjcbg'UbX'Yj U`i Uhjcb"

#### 7 ci fgY 5]a g

H\]g`WcifgY`k]``YbUV`Y`ghiXYbhg`hc`UWei]fY`ZibXUaYbhU``\_bck`YX[Y`cZWUg]b[`XYg][b`UbX`W]b[`Yei]

[ '

## 5ggYgga Ybh

5ggYgga Ybh'HJg_	K Y][ \h	8iY'8UHY	7cifgY`@YUfb]b[ CihWcaYgʻ5ggYggYX
%" <sup>-</sup> 5gg][ ba Ybh%	%\$I	K YY_''	%
&" <sup>-</sup> 5gg][ ba Ybh&	%) I	К ҮҮ_`+	%
' " <sup>:</sup> 5gg][ ba Ybh'	%\$I	K YY_ <sup>:</sup> %\$	&ž''
("`Cb(]bo&16_`]b]h`Ei]n			

Gi Va ]gg]cb bchYg.

## 5hhtybxlubvy Fryeindfya Ybhg

GhiXYbhgʻUfYʻghfcb[`mYbWcifU[YX'hcʻUhhYbX'U```	iXYbhgʻUfYʻg	‰	‰
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 x	-

### FYgci fWg

#### DfYgWf]VYX F Ygci fWyg

Giddofha UhYf]U`Zof'h}g`WolifgY`]bWiX]b[žk\YbYjYf`UjU]`UV`YžWod]Yg`cZ`YWifY`bohYgžj]XYc`W]dgž `YWifY`fYWofX]b[gžfYWola a YbXYX`fYUX]b[gžYhW`WUb`VY`ZoibX`cb`A ccX`Y" H\Y``YWifY`bohY`a Um`VY`j]YkYX`UbX`Xokb`cUXYX`Zfca`h\Y`IBGK!A ccX`Y \hnd.##a ccX`Y"hY`h'ibgk"YXi"Ui#

#### FYWca a YbXYX<sup>-</sup>FYgci fWYg

: c``ck ]b[ g'UfY'h\Y'fYWca a YbXYX'Vcc\_g'Zcf'**7 Ug]b[ '8Yg][ b**.

- F U\a Ub`G"G"`UbX`7 \]`]b[Uf]Ub`; "J ". **7 Ug]b[ `8Yg][ b`H\YcfmUbX`DfUW]V%**"`9`gYj ]Yf`GW]YbW% 6"J "ž`5a ghYfXUa ž`H\Y`BYh\Yf`UbXgž%- ) "
- 6ci f[cmbY'5"H"'>f"žA]`\Y]a '? "? "ž7\YbYjYfhA "9"'UbX'Nci b[ :: "G"'>f". '5dd`]YX'8f]``]b[
  9b[]bYYf]b[žGD9'HYI N/cc\_'GYf]YgžJc`"'&žF]WUfXgcbžHLžI G5ž% %

H\Y`HYI H/cc\_'Zcf'**7 Ya Ybh]b[** ']g.

BY`gcbž'9"6"'UbX'; i]``ch'8"ž'KY``'7% Wa Wbhith ['fGYWcbX'9X]h]cbłž'GW\`ia VYf[Yf

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

H\Y'GW\cc``\Ug'XYjY`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'giVa]hhjb['U'WcifgY'Ugg][baYbh'

K Y YbWci fU[ Y mci 'hc'fYhJ]b'U'WcdmicZYj YfmUgg][ ba Ybhigi Va ]hhYX'Zcf'UggYgga YbhZcf'mci f'ck b'fYWcfX Y]h\Yf']b'\UfXWcdmicf'Y'YWfcb]WZcfa "

5```UggYgga Ybhg`a i gh\Uj Y`Ub`UggYgga YbhWcj Yf`g\YYhUHUWXYX"

## 7cifgY'Wcad`Yh]cb

7 ci fgY Wea d`Yhjcb`fYei ]fYg`gi Va ]gg]cb`cZU```UggYgga Ybh`]hYa g"`: U]`i fY`hc`gi Va ]hU```UggYgga Ybh`]hYa g a UmfYgi `h`]b`h\Y`Uk UfX`cZUb`I bgUhjgZUWcfm: U]`i fY`fI : Ł`[fUXY`Zcf`h\Y`7 ci fgY`i b`Ygg`gdYWjU` Webg]XYfUhjcb`\Ug`VYYb`gi Va ]hhYX`UbX`Uddfcj YX"`D`YUgY`bchYž`U`Wea dYhYbWm\i fX`Y`cZ) \$i ``]g`Udd`]YX`hc h\Y`ZjbU``UggYgga Ybh'

### @UhY Gi Va ]gg]cb cZUb 5gg][ ba Ybh

: i ```a Uf\_g`Zcf`Ub`Ugg][ ba YbhUfY`cb`mdcgg]V`Y`k \Yb`Ub`Ugg][ ba Ybh`]g`fYWY]j YX`Vmh\Y`Xi Y`XUhY''`

K Y'i bXYfgHUbX'h\Uh'Uh'h]a Yg'mci 'a Umbch'VY'UV'Y'hc'gi Va ]h'Ub'Ugg][ ba Ybh'cb'h]a Yž'UbX'h\Y'GW\cc``k ]`` UWWca a cXUhY'UbmZU]f'UbX'fYUgcbUV'Y'YI hYbg]cb"'K Y'k ci `X'fYWca a YbX'mci 'fYj ]Yk 'h\Y'I BGK 'GdYW]U` 7 cbg]XYfUh]cb'[ i ]XY`]bYg'-'gYY'gYWf]cb'VY`ck "

@UhY`gi Va ]gg]cb`k ]```bchVY`UVWYdhYX`UbX`k ]```VY`Wcbg]XYfYX`Ug`bc`gi Va ]gg]cb"

### GdYWJU``7cbg]XYfUh]cb

 $\label{eq:linear_constraint} \begin{aligned} & \mathsf{Nci}^{\mathsf{Wb}} Udd`mZcf`gdYW]U``Wcbg]XYfUH]cb`H\fci [ \`<u>H\Y`Bi WYi g`Gh XYbh<i V</u>`k \Yb`]``bYgg`cf`cH\Yf W]fW a gHUbWfg`]bHYfZYfY`k ]h`mci f`UggYgga YbhdYfZcfa UbWf"`G]W_bYggž`a ]gUXj Ybh fY`cf`cH\Yf W]fW a gHUbWfg`VYmcbX`mci f`Wcbhfc``a Um$ 

- DfYj Ybhimci Zfca 'Wca d'Yhjb[ 'U'Wci fgY'fYei ]fYa Ybh
- ? YYd mci Zfca UthYbX]b[ Ub UggYggUV Y UWjj ]m
- Ghcdimci igi Va ]hh]b[ iUggYggUV'Yik cf\_'Zcf'U'Wci fgY
- G][b]]WUbhmUZYWhinci f`dYfZcfa UbW`]b`UggYggUV`Y`k cf\_žVY`]hU`Zcfa U``YbX!cZgYa YghYf YI Ua ]bUh]cbž`U`WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa ]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa `cZ UggYgga Ybh

K Y`Ug\_`h\Uhimci `d`YUgY`WebhUWih\Y`7 ci fgY`7 cbj Ybcf`]a a YX]UhY`mcbWY`mci `\Uj Y`Wea d`YhYX`h\Y`gdYW]U` Webg]XYfUh]cb`Udd`]WUh]cbžbc``U]\Yf`h\Ub`cbY`k YY\_`Zca `gi Va`]gg]cb"#

‰

## 5WUXYa]W<cbYghmUbX^D`U[]Uf]ga

Mcif`YWMifYfUbX'h\Y'I b]jYfg]mmik]``YIdYWhimcif'giVa]hhYX'Ugg][baYbhg'UfY'hfi`mmcif'ckb'kcf\_"'IBGK

### 5WUXYa]W=bZcfaUh]cb

## 7 ci fgY F Ygi `hg

: cf<sup>·</sup>XYHJ]`gʻcbʻl BGK ʻUggYgga Ybhdc`]Whž'd`YUgYʻj ]g]h'<u>k k k "gh XYbH'i bgk "YXi "Ui #UggYgga Ybh</u>

=ðigca Yi]bgHUbWYgimci fiZ]bU``Wci fgYifYgi `ha UmVYik]h\\Y`X`UbX`bchfY`YUgYX`cbih\Y`I\_BGK\_id`UbbYX`XUhY" H\]gi]gi]bX]WUhYX`VmU`Wci fgYi[fUXYifYgi `hcZY]h\Yf.

- VfcUXVUbX'WcbbYWfjcb'f&) \* '\_V]h#gYWcf'ZJghYfŁ
- UV]`]hmhc`j`]Yk`ghfYUa`]b[`j`]XYc`fN][\`cf``ck`XYZ]b]h]cb`l\_BGK``HJ`cdh]cbgŁ

A cfY`]bZcfa Uh]cb'UVci hgmghYa 'fYei ]fYa Ybhg']g'Uj U]`UV`Y Uh<u>k k k "ghi XYbh'i bgk "YXi "Ui #a ccX`Y!gmghYa !fYei ]fYa Ybhg</u>

## 5\WWgg]b['7cifgY'AUHYf]U`g'H\fci[\'AccX`Y

7 ci fgY ci h]bYgžgi ddcfha UhYf]U`gʻUfYʻi d`cUXYX`hc`A ccX`Yžh\Y`i b]j Yfg]hnighUbXUfX`@YUfb]b[ A UbU[ Ya YbhGmghYa `f@A GŁ"`±b`UXX]h]cbžcb!`]bY`Ugg][ ba Ybhigi Va ]gg]cbg`UfY`a UXY`i g]b[ `h\Y Ugg][ ba YbhXfcdVcI `ZJVJ`]hnidfcj ]XYX`]b`A ccX`Y"`5```Ybfc``YX`ghi XYbhg`UfY`Ui hca Uh]WU``m`]bWi XYX`]b  $\label{eq:linear} \begin{array}{l} K_{\rm c}^{\rm T}UW_{\rm b}ck_{\rm c}^{\rm T}X[Y^{\rm t}XY^{\rm t}GYXY[U^{\rm t}dYcd^{\rm T}Y^{\rm t}k_{\rm c}^{\rm t}UfY^{\rm t}XY^{\rm t}fUX]h]cbU^{\rm t}W_{\rm c}Dg^{\rm t}cZh_{\rm c}^{\rm T}UbXg^{\rm t}cb^{\rm t}k_{\rm c}^{\rm t}WX^{\rm t}BGK_{\rm c}^{\rm t}Ybg]b[hcb^{\rm t}Wa_{\rm d}i_{\rm g}^{\rm t}]g^{\rm t}cWW_{\rm c}^{\rm t}X''$ 

## 5ddYbX]I. '9b[]bYYfg'5ighfU`]U'f195ŁDfcZYgg]cbU`'9b[]bYYf'7cadYhYbWm GHJbXUfX

Dfc[fUa +bhybXYX @YUfb]b[ Cihwca Yq ? bck `YX[ Y 'UbX'q\_]``'VUqY D9%"%7cadfY\Ybq]iYžh\YcfmVUqYXibXYfqHUbX]b[cZh\YibXYfd]bb]b[bUhifU`UbX d/mg]WU`:gW]YbWYg`UbX`h\Y`Yb[]bYYf]b[`Z bXUa YbHU`g`Udd`]WUV`Y`hc`h\Y`Yb[]bYYf]b[`X]gW]d`]bY D9%"& 7 cbW/dhiU`i bXYfqHUbX]b[ cZh\Y`a Uh\Ya Uh\Wažbi a Yf]WU`UbU`mg]qžqHUh|qh\WažUbX Weadi hYf'UbX']bZcfaUh]cb'gW]YbWYg'k\]WX'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW[d`]bY D9%" + + XYdh i bXYfqHbX]b[ cZqdYWU`]qhVcX]YqcZ bck YX[Yk]h bhY'Yb[]bYYf]b[ X]qWd`]bY D9%"('8]qWFfba YbhcZ\_bck YX[YXY]Ycda YbhUbXfYqYUfWXX]fYWfcbg'k ]h.jb'h.Y Yb[]bYYf]b[`X]gWJd`]bY D9%") ? bck `YX[Y`cZYb[]bYYf]b[ `XYg][b`dfUW]WY`UbX`WcbhYl hi U``ZJW/cfg`]a dUWf]b[ `h\Y Yb[]bYYf]b['X]qW[d`]bY D9%"\* I bXYfqHJbX]b[ cZh\Y'qWcdYždf]bW]dYgžbcfa gžUWVci bHJV]]]h]Yg'UbX'Vci bXg'cZ qi qHU]bUV`Y`Yb[]bYYf]b[`dfUWf\WY`]b`H\Y`qdYW[Z]WX]qW[d`]bY 9b[]bYYf]b['Udd`]WUh]cb'UV]`]m D9&"%5dd`]WUhjcb`cZYghUV`]g\YX'Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`YI 'Yb[]bYYf]b[`dfcV`Ya qc`i]b[ D9&"&: `i YbhUdd`]WUhcb'cZYb[]bYYf]b[ 'hYWkb]ei Ygžhcc`g'UbX'fYgci fWfg D9&" 5dd`]WUhjcb`cZgmghYa UhjWYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg D9&"( '5dd`]WUHcb'cZgmghYa UHWUddfcUWXYg'hc'h\Y'WcbXi WiUbX'a UbU[Ya YbhcZYb[]bYYf]b[ dfc Y₩q DfcZYgg]cbU`UbX dYfgcbU`Uhf]Vi hYg D9' "%'9h\]WU``WcbXi WiUbX'dfcZygg]cbU``UVWci bHUV]`]hm D9' "&'9ZYW) Y'cfU`'UbX'k f)HYb'Wca a i b)WU) cb']b'dfcZygq]cbU`'UbX'`UmXca U]bg D9' "' 7 fYUhi Yžibbci Uhi YUbX dfc!UWi YXYa YUbci f D9' "( DfcZygg]cbU`i gY'UbX'a UbU[ Ya YbhcZ]bZcfa Uhcb D9' ") CfXYfma UbU[Ya YbhcZgYZzUbX dfcZygq]cbU`WcbXi Wh D9' "\* 9ZYWAJ Y HYUa 'a Ya VYfg\]d UbX HYUa 'YUXYfg\]d