# **PTRL3040**

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# 7 ci fgY<sup>-</sup>Cj Yfj ]Yk

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#### 7cbj Ybcfg

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han

\_bck `YX[Y'UbX`YI dYf]YbW"''=b`dfUWf]WfžUZłYf`XY`]j Yf]b['Ub']a dcffUbhhcd]W#WcbWYdhžk Y`k ]``[]j Y`ghi XYbhg cddcfhi b]hmhc`X]gW gg'h\Y'hcd]WWcj YfYX''D=dfcj ]XYg'Ub'cddcfhi b]hmZcf`ghi XYbhg`hc`WUf]Zmh\Y]f`ck b i bXYfgfUbX]b[']b'U'WcbhYI hh\Uh]g``Ygg'WcbZfcbh]b['h\Ub'X]fYWhmUbgk Yf]b['ei Ygh]cbg'Zfca 'h\Y']bghfi Wfcf" H\]g'UddfcUVX 'Vi ]`Xg'h\Y``YUfb]b['Wca a i b]hmUg'ghi XYbhg`fYU`]nY'h\YmUfY'bchU`cbY`]b'a ]gi bXYfgfUbX]b[ cf'dUfh]U``mWca dfY\YbX]b['\_\_YmWcbWYdhgžUg'k Y```Ug'dfcj ]X]b['ZYYXVUW\_'hc'h\Y'=bghfi Wfcf'cb'h\Y'WUgg Wca dfY\Ybg]cb''

#### Hihcf]U`!@YWhifY`Z]dd]b[

Ghi XYbhg UfY WLU`Yb[YX'hc igcij Y'U'hi hcf]U`iei Ygh]cb VYZcfY'h Y'fY`UhYX'hcd]WLUg VYYb'X]gW ggYX']b'h Y`YWhi fY'''H\Y'ghi XYbhg'k]``k cf\_']b'[fci dg'': ]fghih Ymk]``X]gW gg'h Y'dfcV`Ya `ghUhYa Ybhk ]h 'h Y]f'dYYfg`UbX']XYbh]ZmhNY'dfcV`Ya '''Hi hcfg'k]``[YhZYYXVUW\_Zfca 'j Uf]ci g'[fci dg'UbX'[i]XY'h Ya ']Zh Y]f'dfcV`Ya[XYbh]Z]WUh]cb `]g']bWcffYWh'5ZhYfk UfXgž'h Ymk]```]XYbh]Zmdcgg]V'Y gc`i h]cb 'a Yh\cXg'UbX'Z]bU`mgc`j Y'h YdfcV`Ya '''5[U]bž'h hcfg'k]```]bWfYa YbhU`m[i]XY'ghi XYbhg'h fci [\'fh Y'gc`i h]cb 'cZh Y'dfcV'Ya '''H\YWcbWi g]cbg'XfUk b'Zfca 'h\Y'h hcf]U`U'][b')WWc'%o</

# 5HYbXUbWFFYei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`mYbWci ffYbhgʻ



#### FYgci fWfg

#### DfYgWf]VYX F Ygci fWyg

BchUjU]`UV`Y

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#### 7 ci fgY 9j U`i Uhjcb UbX 8Yj Y`cda Ybh

ww

5 htty YbX cZYUW. Wei fgYžU```ghi XYbhg`k]```\Uj Yftty cddefhi b]hmhe Wea d`YhY U'Wei fgY Yj U`i Uhjeb Zefa " H\YgY`Ubebma ei g`gi fj Ymg`\Y`d`i g`i bXYfgHUbX`mei f`j ]Yk g`eZhty`Wei fgYžmei f``YWhi fYfg`UbX`hty`Wei fgY a UhYf]U`g"`K Y`UfY`Webhjbi ei g`m`]a dfej ]b[ `ei f`Wei fgYg`VUgYX`eb`ghi XYbhZYYXVUW\_ž'UbX`mei f`dYfgdYWhj Y ]g`j U`i UV`Y"

: YYXVUW\_`]g`[]j Yb`j]U`<u>\hhdg.##gh XYbh'i bgk "YXi "Ui #a m¥ siYf]YbW</u>`Uby mci `k]``VY`bch]Z]YX`k \Yb`h\]g`]g Uj U]`UV`Y`Zcf`mci `hc`Wca d`YhY"

K Y'U`gc'YbWci fU[Y'U```ghi XYbhg'hc'g\UfY**UW**mZYYXV**ew** XW'\Uj Y'Ubmih]a Y'Xi f]b['h\Y'Wci fgY'-']Zmci \Uj Y'U'WcbWfbž'd`YUgY'WcbHUWii g']a a YX**UW**(m

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

H\Y'GW\cc``\Ug'XYjY`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'giVa]hjb['U'WcifgY'Ugg][baYbh'

K Y`YbWci fU[Y`mci `hc`fYHJ]b`U`WcdmicZYj YfmUgg][ba Ybhigi Va ]HYX`Zcf`UggYgga YbhZcf`mci f`ck b`fYWcfX Y]h\Yf`]b`\UfXWcdmicf`Y`YWfcb]WZcfa "

5```UggYgga Ybhg`a i gh\Uj Y`Ub`UggYgga YbhWcj Yf`g\YYhUhUWXYX"

## 7cifgY<sup>·</sup>Wcad`Yh]cb

7 ci fgY Wea d`Yh]cb`fYei ]fYg`gi Va ]gg]cb`cZU```UggYgga Ybh`]hYa g":: U]`i fY`hc`gi Va ]hU```UggYgga Ybh`]hYa g a UmfYgi `h`]b`h\Y`Uk UfX`cZUb`I bgUh]gZUWcfm:: U]`i fY`fI : Ł`[fUXY`Zcf`h\Y`7 ci fgY`i b`Ygg`gdYW]U` Webg]XYfUh]cb`\Ug`VYYb`gi Va ]hhYX`UbX`Uddfcj YX"`D`YUgY`bchYž'U`Wea dYhYbWn\i fX`Y`cZ) \$i ``]g`Udd`]YX`hc h\Y`Z]bU``UggYgga Ybh'

### @UhY Gi Va ]gg]cb cZUb 5gg][ ba Ybh

: i ```a Uf\_g`Zcf`Ub`Ugg][ ba YbhUfY`cb`mdcgg]V`Y`k \Yb`Ub`Ugg][ ba•wbh]g`TYWY]j YX`Vmh\Y`Xi Y`XUhY"`

K Yʻi bXYfgHUbXʻh\UhUhihja Ygʻmci 'a Umbch, YʻUV`Yʻhc<sup>o</sup>y, Xa jhUbʻUgg][ ba Ybhicbʻhja YžUbXʻh\YʻGW\cc``k ]`` UWVca a cXUhYʻUbm'zu]fʻUbXʻfYUgcbUV YʻYI Kovycb" K, Yek ci `XʻfYVca a YbX'mci ʻfYj ]Yk ʻh\YʻI BGK ʻGdYWJU` 7 cbg]XYfUhjcbʻ[ i ]XY`]bYgʻ-ʻgYYvyYVVDbyVY`c

@UhY`gi Va ]gg]cb`k\_]``bchVY\*LVWY#hX;UbX`k\_]``VY`Wcbg]XYfYX`Ug`bc`gi Va ]gg]cb"

## GdYWJU`7cbg]XXfUhjcb

 $\label{eq:linear_line$ 

- DfYj Ybhmci Zfca Wca d`Yhjb[ U'Wci fgY'fYei ]fYa Ybh
- ?YYd'mci 'Zfca 'UHYbX]b[ 'Ub'UggYggUV'Y'UWfj ]hm
- Ghcdimci igi Va ]hh]b[ iUggYggUV'Yik cf\_'Zcf'U'Wci fgY
- G][ b]Z]WUbhmUZYWhinci f`dYfZcfa UbW'`]b`UggYggUV`Y`k cf\_ž`VY`]hU`Zcfa U``YbX!cZgYa YghYf YI Ua ]bUh]cbž`U`WUgg`hYghž`U``UVcfUhcfmhYghž`U`gYa ]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa `cZ UggYgga Ybh

K Y`Ug\_`h\Uhimci `d`YUgY`WcbhUWih\Y`7 ci fgY`7 cbj Ybcf`]a a YX]UhY`mcbWY`mci `\Uj Y`Wca d`YhYX`h\Y`gdYW]U` Wcbg]XYfUh]cb`Udd`]WUh]cbžbc``UhYf`h\Ub`cbY`k YY\_`Zca `gi Va ]gg]cb"

A cfY`XYHJ]`g`cb`gdYWJU``Wcbg]XYfUHjcb`WUb`VY`Zci bX`Uh`<u>k k k "ghi XYbH'i bgk "YXi "Ui #gdYWJU`!Wcbg]XYfUHjcb</u>

# Gh XYbhGi ddcfh

H\Y`I\_b]j Yfg]miUbX`H\Y`: UW`hmdfcj ]XY`U`k ]XY`fUb[Y`cZgi ddcfhgYfj ]Wg`Zcf`gh XYbhgž`]bWi X]b[.

- | BGK "@YUfb]b[ "7YblfY"! "<u>k k k "`Wi bgk "YXi "Ui</u>
- 7 ci bgY``]b[ 'gi ddcfh! <u>k k k "Wei bgY``]b[ "i bgk "YXi "Ui</u>

9ei ]HUV`Y`@YUfb]b[ `GYfj ]WYg`U]a g`hc`dfcj ]XY`U```ghi XYbhg`k ]h\`U`ZYY`UbX`WcbZXYbh]U``gYfj ]WY`h\Uh

# 5WUXYa]W<cbYghmUbX^D`U[]Uf]ga

Mcif`YWMifYfUbX'h\Y'Ib]jYfg]mik]``YIdYWhimcif'giVa]hhYX'Ugg][baYbhg'UfY'hfi`mimcif'ckb'kcf\_"'IBGK





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 $\label{eq:linear} \begin{array}{l} K_{1}^{*}(UW\underline{b}ck^{*}YX[Y^{*}hY^{*}GYXY[U^{*}dYcd^{*}Y^{*}k \setminus c^{*}UfY^{*}hY^{*}fUX]h]cbU^{*}WghcX]Ubg^{*}cZhY^{*}UbXg^{*}cb^{*}k \setminus ]W_{1} \mid B\,GK \mid P_{2}^{*}(Wdy^{*}hY^{*}h$ 

# 5ddYbX]I. '9b[]bYYfg'5ighfU`]U'f95E'DfcZYgg]cbU`'9b[]bYYf'7cadYhYbWm GHJbXUfX

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?bck`YX[Y'UbX'g_]``VUgY	
D9%"%"7cadfY\Ybg]jYz"h\YcfmVUgYX"ibXYfgHUbX]b["cZh\Y"ibXYfd]bb]b["bUhifU``UbX d\mg]WU``gW]YbWYg`UbX"h\Y`Yb[]bYYf]b["ZbXUaYbHU`g`Udd`]WUV`Y"hc"h\Y`Yb[]bYYf]b["X]gW]d`]bY	
D9%"&`7cbWYdhiU``ibXYfghUbX]b[`cZh\Y`aUh\YaUh]WgžbiaYf]WU``UbU`mg]gžghUh]gh]WgžUbX WcadihYf`UbX`]bZcfaUh]cb`gW]YbWYg`k\]W\`ibXYfd]b`h\Y`Yb[]bYYf]b[`X]gW]d`]bY	
D9%" `=Ð!XYdh\`i bXYfghUbX]b[ `cZgdYW]U`]ghVcX]Yg`cZ_bck `YX[ Y`k]h\]b`h\Y`Yb[]bYYf]b[ X]gW]d`]bY	
D9%"( `8]gWYfba YbhcZ_bck `YX[ Y`XYj Y`cda YbhUbX`fYgYUfW\ XYYWjcbg`k ]h\]b`h\Y Yb[]bYYf]b[ `X]gW]d`]bY	
D9%") '? bck `YX[Y`cZYb[]bYYf]b[`XYg][b`dfUWf]WY`Uby YcbHYI h U``ZUVFfg`]a dUWf]b[`H\Y Yb[]bYYf]b[`X]gW]d`]bY	
D9%"* `I_bXYfgHJbX]b[ `cZh\Y`gVcdYždf]b <b>y</b> ld Ygžbcf <b>aar</b> wWci_bHJV]`]h]Yg`UbX`Vci_bXg`cZ gi_gHJ]bUV`Y`Yb[ ]bYYf]b[ `dfUW]WY`]b`h\Y`g <b>d w1</b> []WX]gWd"]bY	
9b[]bYYf]b[`Udd`]WUh]cb`UV]`]m	
D9&"%5dd`]WUHjcbicZYgHUYaqYXXa[]bYYf]b[`aYH\cXg'hc'Wcad`YI`Yb[]bYYf]b[`dfcV`Ya gc`j]b[	
D9&"&`: `i YbhUdd`]WUheb`czYb[`]bYYf]b[ `hYW\b]ei Ygžhcc`g`UbX`fYgci fWfg	
D9&"'`5dd`]WUHjcb`cZgmgHYaUHjWYb[]bYYf]b[`gmbH\Yg]g`UbX`XYg][b`dfcWfggYg	
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DfcZYgg]cbU``UbX`dYfgcbU``UHf]Vi HYg	
D9' "%'9h\]WU``WebXi WhUbX`dfcZYgg]cbU``UWWei bhUV]`]hm	
D9' "&`9ZXWMj Y`cfU``UbX`kf]mYb`Wcaaib]WUhjcb`jb`dfcZYgg]cbU``UbX``UmXcaU]bg	
D9' ''' '7 fYUhji Yži]bbcj Uhji YʻUbXʻdfc!UWhji YʻXYa YUbci f	
D9' "( `DfcZygg]cbU``i gy`UbX`a UbU[ Ya YbhcZ`]bZcfa Uh]cb	
D9' ") °C fXYfma UbU[ Ya YbhcZgY`Zz̈UbX`dfcZygg]cbU``WcbXi Wh	
D9' "* '9ZYWNjj Y NYUa 'a Ya VYfg\]d UbX NYUa ``YUXYfg\]d	