

CVEN4201

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HYfa 'C bY''##'&\$&%

7ci fgY Cj Yfj]Yk

GHJZ:7cbHJWh'8YHJ]`g

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GWkcc~7cbhJWh~abZcfa Uh]cb

<u>9b[]bYYf]b['Ghi XYbhGi ddcfhGYfj]WYg'-</u>'H\Y'Bi WYi g'!'Ybfc`a Ybhž'dfc[fYgg]cb'W\YW_gž'WUg\'fYei Yghgž Wti fgY']ggi Yg'cf'dfc[fUa!fY`UhYX'ei Yf]Yg'

9b[]bYYf]b[*\DXi glf]U``HfU]b]b[-- \DXi glf]U``lfU]b]b['ei Ygl]cbg'

<u>I_BGK_'Gh_Xm5VfcUX</u>'-gh_XmUVfcUX'gh_XYbhYbei_]f]Yg_fZcf']bVci_bX'gh_XYbhgŁ'

<u>I BGK '91 WXUb[Y</u>'-gh XYbhYl WXUb[Y'Ybei]f]Yg'flcf']bVci bX'gh XYbhgl'

LBGK : i h fY Gh XYbh - dch bh U`gh XYbh Ybei]f]Yg Y"["UXa]gg]cbgž ZYYgž dfc[fUa gž WfYX]h hfUbgZYf

D\cbY.

fŽ*%&Ł-',),)\$\$ - Bi WYi g Gh XYbh<i V

fŽ*%%&L'-',) '+**%'-'9b[]bYYf]b['=bXi ghf]U`'HfU]b]b['

fŽ*%%&£'-',) " %+- '-1 BGK 'Gh Xm5VfcUX'UbX'1 BGK '91 WXUb[Y'fZcf']bVci bX'gh XYbhg£'

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

8 YgWf]dh]cb cZfcW_a Ugg UbX X]gWcbh]bi]h]Yg/fcW_ghfYb[h\ UbX ZJ]`i fY Wf]hYf]U" 7 cfY `c[[]b[/Z]Y`X XUHU Wc``YWf]cbza Udd]b[UbX ZfUWh fY gi fj Ymg/XUHU dfYgYbHUh]cb/\Ya]gd\Yf]WU` dfc YWf]cbg/]bhfcXi WcfmfcW_g`cdY gHUV]`]hm/Zci bXUh]cbg cb fcW_/YI WUj Uh]cb cb fcW_/]b!g]hi ghfYgg/ghfYggYg Uvci hi bXYf[fci bX cdYb]b[g/WUgg]Z]WUh]cb gmghYa g UbX hi bbY``gi ddcfhfYei]fYa Ybhg/g]hY]bj Ygh][Uh]cbg Zcf``UbXg`]XYg UbX g`cdY gHUV]`]guh]cb hYW b]ei Yg/i gY cZg`cdY gHUV]`]hmUb`Umg]g dfc[fUa g"

7ci fgY 5]a g

 $\label{thm:local_problem} He `YbUV`Y`mci `hc`VY`UV`Y`hc`dYfZcfa `VUg]WXYg][bg`]b`feW_`]bWi X]b[`Zci bXUh]cbgž'g`cdY`ghUV]`]hmUbX hi bbY``XYg][bg''$

Hc'ghi Xmh\Y'VUg]Wdf]bV]d\Yg'fY`UhYX'hc'h\YcfmUbX'XYg][b'cZfc_'UbX'gc]`'g`cdYg']bWi X]b['Ub Yi Ua]bUh]cb'cZh\Y'X]ZZYfYbhimdYg'cZg`cdY']bghUV]`]mUbX'X]ZZYfYbhig]hY']bj Ygh][Uh]cb'a Yh\cXg'za Yh\cXg'cZ UbU`ng]b['g`cdYg'UbX'X]ZZYfYbhia Yh\cXg'Zcf'ghUV]`]g]b['g`cdYg''

Hc'[]jY'mci 'gca Y'Yl dYf]YbWY']b'i g]b['Wca di hYf'gcZtk UfY'hc'UggYgg'h\Y'ghUV]`]hmicZU'g`cdY'UbX'j Uf]ci g fYa YX]Uh|cb'a YUgi fYg"

Hc [] Y mci 'U'dfUW]WU`i bXYfgrUbX]b['cZfcW_'UbX'g'cdY'Yb[]bYYf]b["

7cifgY@YUfb]b[CihWcaYg

52AYfigi WWggZi ``mWa d`Yh]b['h\]giWa fgYžmci ig\ci `X'VY'UV`Y'hc.

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%"6Y`ZJa]`]Uf`k]h\`h\Y`VUg]Wdf]bW]d`Yg`fY`UhYX`hc`h\Y`h\YcfmUbX XYg][b`cZfcW_Yb[]bYYf]b[`]bWiX]b[`aYh\cXg`Zcf`XYgWf]V]b[ž fYWcfX]b[`UbX`dfYgYbh]b[`ZYUhifYg`cZfcW_aUggYg	D9%"% D9%" žD9%"(žD9&"% D9&"(žD9' "*	
&"`Hc`VY`UV`Y`hc`dYfZcfa `VUg]WXYg][bg`]b`fcW_`]bWi X]b[D9%"%žD9%"&žD9%" žD9%") ž	
Zci bXUh]cbgž'g`cdY`gHJV]`]hmUbX`h bbY``XYg][bg	D9&"%žD9&"&žD9&"'	
' "'6Y'ZJa]`]Uf'k]h\'h\Y'VUg]Wdf]bW]d`Yg'fY`UhYX'hc'h\Y'h\Ycfm'UbX	D9%"%žD9%"&žD9%" žD9%") ž	
XYg][b'cZfcW_'UbX'gc]`'g`cdYg	D9&"%žD9&"&žD9&"'	
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UbX'fcW_'g`cdYg'UbX'X]ZYfYbha Yh\cXg'Zcf'ghJV]`]g]b['h\Ya"	D9&"%ž"D9&"'	
) "'6Y'UV'Y'ho'i gY'h\Y'goZhk UfY'dUW_U[Y'G`cdYK 'ho'UggYgg'h\Y	D9%" ž'D9%") ž'D9&"%ž'D9&"&ž	
ghUV]`]hmcZU'g`cdY'UbX'j Uf]ci g'fYa YX]Uh]cb'a YUgi fYg"	D9&"' ž'D9%"&	
*"'Hc'\UjY'U'dfUWjWU`i bXYfgHJbX]b['cZfcW_'UbX'g`cdY	D9' "&ž'D9' "(ž'D9&"%ž'D9&"&ž	
Yb[]bYYf]b['i g]b['H\Y'Ugg][ba Ybhg"	D9&"' ž'D9&"(ž'D9%")	
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HYUWX]b['GhfUhY[]Yg'

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@YWNifYg'k]```j UfmZfca 'ghUbXUfX'Dck YfDc]bh``YWNifYg'nc'a cfY'\UbXg'cb'XYa cbghfUh]cbg'cZj Uf]ci g
Yb[]bYYf]b['h\YW,b]ei Yg'''Mci 'UfY'Yl dYWMYX'nc'j]Yk 'U``'h\Y``YWNifYg'`]j Y'Ug'h\Ymk]```[fYUhmUgg]ghi]b
i bXYfghUbX]b['k\Uhi]g'dfYgYbhYX']b'h\Y'`YWNifY'bchYg"'H\Y``YWNifYg'k]``U'gc'VY'U'df]a Ufmdc]bhcZ
Wcaaib]WUh]cb'VYhkYYb'h\Y'WUgg'UbX'amgY'Z': ifh\Yf'Wcaaib]\WUh]cb'k]``VY'j]U'mcif'ghiXYbhYaU]`'UbXAcxY''=i]g'jYfm]adcfhUbh'h\Uhmci'ZfYeiYbhmW\YW_mcif'aYggU[Yg"

8Ya cbglfUljcbg#k cf_g\cdg. 'H\Y`XYa cbglfUljcbg#k cf_g\cdg`]b`l\]gʻgi VYWliUfY`i gYX`lc`lYUW\`mci '\UbXg cb'fcW_XYgWljdljcb`UbX`WUgg]Z\Wljcb/d`chjb[cZXYZYWliXUllU`UbX`i gY`cZWca di lYf`dfc[fUa g"`H\Ymk]``

5ggYgga Ybh

h\Y'Ugg][ba Ybhk]``'VY'gd`]h]bhc'Wca dcbYbhg'k]h\'Xi Y'XUhYg'X]ghf]Vi hYX'h\fci [\ci hh\Y'gYgg]cb"'H\]g'k]``U``ck 'mci 'hc'Wca d'YhY'YUW\'dUfh'Ug']h]g'hUi [\hi]b'WUgg'fg]a]`Uf'hc'U'k cf_g\cdb'gc'Ug'bchhc'cj Yf'cUX mci fgY'j Yg'Uhh\Y'YbX'cZh\Y'gYgg]cb'UbX'hc'YbUV'Y'a Y'hc'[]j Y'mci 'gca Y'a Uf_g#ZYXVUW_'df]cf'hc'h\Y'YbX cZh\Y'mYUf''=f]g'U]a YX'hc'[]j Y'ZYYXVUW_'k]h\]b'hk c'k YY_g'cZgi Va]gg]cb"

@UhY`gi Va]gg]cbg`k]```VY`dYbU`]gYX`Uh'h\Y`fUhY`cZ%\$i `dYf`XUmUZhYf`h\Y`Xi Y`h]a Y`UbX`XUhY`\Uj Y`YI d]fYX

5qqYqqa Ybh'& 'G`cdY'GhUV]`]hm

GHUfh'XUhY. K YY (

8YW]`q.

H\]gʻUgg][ba Ybhk]``Wcj Yfʻh\YʻgʻcdYʻgʻtUV]`]miVca dcbYbhcZh\YʻWci fgY"'±ik]``UggYggʻmci fʻUV]`]mihc
UbU`mgY`UʻgʻcdYʻi g]b['h\Yʻdfc[fUa 'GʻcdY#K 'UbX'hc`Wcbg]XYfʻUbX`XYg][bʻdcgg]V`YʻfYa YX]Uf]cbʻa YUgi fYg"
; YchYW\b]WU``Yb[]bYYfgʻcZhYbʻi gY`Wca di hYfgʻhc`XYg][bʻUbX`UbU`mgYʻgʻcdYgʻgc']h`]gʻ]a dcfhUbhih\Uhmci
`YUfb`\ck 'hc'i gY`h\Ya 'dfcdYf`mUbX`U`gc'j Yfm]a dcfhUbhim`YUfbʻh\Y]f``]a]hUf]cbg"'E i U`]hmcZdfYgYbhUf]cb']g
U`gc']a dcfhUbh'

@UhY`gi Va]gg]cbg`k]```VY`dYbU`]gYX`Uh'h\Y`fUhY`cZ%\$i `dYf`XUmUZhYf`h\Y`Xi Y`h]a Y`UbX`XUhY`\Uj Y`YI d]fYX

5ggYgga Ybh' . :]bU`Yl Ua

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &\cifgfHchU\h]a Y]bWi X]b[fYUX]b[#d\cUX\H67\end{a}

8YHJ]`g.

 $\label{thm:condition} $$HY'Z_bU''CD9B'6CC?'YIUa''k_]^*UggYgg'mcif'ibXYfgHbX]b['cZh\Y'k_\c'Y'WcifgY'k_]h''dUffHW''Uf Yad\Ug]g'cb'mcif'UV]]$$ $$Imhc'gmbh\Yg]gY'XUHU'UbX']bjYgh[[UhY'UbX'XYg][b'ghfiWhfYg']b'fcW_aUggYg'UbXg'cdYg''5'aUf_'cZUh'YUgh'($i ']b'h\Y'Z_bU''YIUa]bUh]cb']g'fYei]fYX'VYZcfY'h\Y'WUgg'kcf_']g']bWiXYX']bh\Y'Z_bU''aUf_$

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg

H\Y`GW\cc``Wca di hYf``UVcfUhcf]Yg`dfcj]XY`mci `UWW\gg`hc`DaUeU[W`UWdfc[fUa gž6 [be`fd`chh]b[`XYZYWh XUhU\L'UbX`7j S_ [` WUbX`DE \$`fl bXYf[fci bX`ghfYgg`UbU`mg]g\L'UbX`h\Y`9 WaE fgV[a`dfc[fUa `9 WaE 'abWfgc]` g`cdY`ghUV]`]hm\UbU`mg]g\L''`GcZk UfY`WUb`U`gc`VY`UWW\ggYX`j]U`3UUW\e 3` kf[_ W3` ki Z\W\U': cf`Yl Ua d`Y`Zcf` DaUeU[W`U\WgcZk UfY.`\hdg.#k k k "a m\U\W\gg"i bgk "YXi "Ui #Udd`]\\Uhjcbg#c\\U\Y\V\!gi]hY

7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y'cda Ybh

 $H Y W i fg Y] g Y Y Y U b b i U mh f c i [\h Y a m9 I d Y f] Y b W g i fj Y m' 5 ``f Y g d c b g Y g U f Y W b g] X Y f Y X U b X = a U_Y W U b [Y g h c h Y W i fg Y U b b i U m] b f Y g d c b g Y " = U a U g c U k U m g \ U d d m h c [Y h Z Y X V U W_ X i f] b [h Y W i fg Y Z c f] a a Y X] U h Y W b g] X Y f U h j c b " N c i W b Y a U] ``a Y X] f Y W m c f i g Y h Y Z Y X V U W_ X] g W g g] c b h c c `` = \ U j Y d ``U W X] b A c c X Y "$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

 $D`YUgY`fYZYf`hc`h\Y`A~ccX`Y`dU[~Y`cZh\Y`Wci~fgY`Zcf`Z~fh\Yf`[~i~]XUbWY`cb`UggYgga~Ybhigi~Va~]gg]cb"$

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

6Yk UfY "5 b 'Ugg][ba Ybhih Uh]bWi XYg 'd `U[]Uf]gYX 'a UhYf]U` 'k]```fYWY]j Y 'U`\$ı : U]`ž 'UbX 'ghi XYbhg 'k \c d`U[]Uf]gY 'UfY 'U`gc '`]UV`Y 'hc 'X]gW]d`]bUfm'UWf]cbž]bWi X]b[YI Wi g]cb 'Zfca 'Ybfc`a Ybh'

D`U[]Uf]ga `]g'h\Y'i gY'cZUbch\Yf'dYfgcb'g'k cf_cf']XYUg'Ug`]Zh\Ymk YfY'mci f'ck b"'K \Yb`]h]g'bYWfggUfmcf XYg]fUV`Y'hc'i gY'ch\Yf'dYcd`Y'g'a UhYf]U``mci 'g\ci `X'UXYei UhY`mUW_bck `YX[Y'k \cgY'k cfXg'cf']XYUg'h\Ym UfY'UbX'k \YfY'mci 'Zci bX'h\Ya 'f[]j]b['h\Y'Wta d`YH'fYZYfYbW'XYhU]`gž']bWi X]b['dU[Y'bi a VYfffy±"'H\Y

5WUXYa]W±bZcfa Uh]cb

<u>? Ymil_BGK_'8 UhYg</u>'! 'Y["'7 Ybgi g'8 UhYžYl Ua_'XUhYgž``UghiXUmhc 'Xfcd'U'Wci fgY'k]h\ci h'UWUXYa]\\#ZbUb\\JU\`\]UV\]\m\Y

:]bU`'9l Ua]bUh]cbg.

:]bU`YI Ua g`]b'HYfa '%k]``VY`\Y`X`cb`]bY`VYfk YYb'' h '5df]`!'% h 'A Um]bWi g]j Y''Mci 'UfY'fYei]fYX`fc 'VY Uj U]`UV`Y`cb'h\YgY`XUfYg"'D`YUgY`Xc'bchhc'a U_Y`UbmdYfgcbU``cf'hfUj Y``UffUb[Ya Ybhg'Xi f]b['f\]g'dYf]cX"

Gi dd Ya YblUfm9l Ua]bUh]cbg.

Gidd`Ya YbhUfm9I Ua]bUh]cbg`Zcf`HYfa '%'&\$&%k]``VY`\Y`X`cb`&(h\`!`&, h\`A Um]bWig]j Y`g\ci``X'mci 'VY fYei]fYX`hc`g]hcbY"'Mci 'UfY`fYei]fYX`hc`VY`Uj U]`UV`Y`cb`h\YgY`XUhYg"'D`YUgY`Xc`bchhc`a U_Y`UbmdYfgcbU` cf`hfUj Y``UffUb[Ya Ybhg`Xi f]b['h\]g`dYf]cX"

5ddYbX]l. '9b[]bYYfg'5i glfU`]U'f95Ł'DfcZYgg]cbU`'9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa '±bHYbXYX'@YUfb]b['CiHWca Yg	
?bck `YX[Y'UbX'g_]```VUgY	
D9%%7ca dfY\Ybg]j Yžh\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhU`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY	
D9%"&"7cbWfdhiU``ibXYfghUbX]b['cZh\Y'aUh\YaUh]WgžbiaYf]WU``UbU`mg]gžghUh]gh]Wgž'UbX WcadihYf'UbX']bZcfaUh]cb'gW]YbWYg'k\]W\ibXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '±b!XYdh\'i bXYfgHJbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y`k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%'('8]gWYfba YbhcZ_bck `YX[Y`XYj Y`cda YbhUbX`fYgYUfW\ X]fYWIjcbg`k]h\]b`h\Y Yb[]bYYf]b['X]gWJd`]bY	
D9%) '? bck `YX[Y`cZYb[]bYYf]b['XYg][b'dfUWf]WY'UbX'WcbhYl hi U``ZUWcfg']a dUWf]b['h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y 'gWcdYždf]bW]d`Ygžbcfa gž'UWWci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWf]WY']b'h\Y 'gdYW]ZJWX]gW]d`]bY	
9b[]bYYf]b['Udd']WUh]cb'UV]']m	
D9&"%5dd`]\Wlr]cb`cZYgHJV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYWXb]ei Ygž'hcc`g'UbX'fYgci fWfg	
D9&"' '5dd`]WUh]cb'cZgmghYa Uh]WYb[]bYYf]b['gmbh\Yg]g'UbX'XYg][b'dfcWYggYg	
D9&"('5dd`]\WJr]cb`cZgmghYaUh]\WUddfcU\\XYg`hc`h\Y`\\cbXi\WiUbX`aUbU[YaYbhcZYb[]bYYf]b[dfcY\\mathred{Wg}	
DfcZYgg]cbU``UbX`dYfgcbU``Ultf]Vi H/g	
D9' "%'9h\]WU``W&bXi WhUbX`dfcZYgg]cbU``UVW&i bHJV]`]hm	