

**The valuation and assessment
of retirement income products:
A unified Markov chain Monte
Carlo framework**



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Overview

- **Motivation**
- **The multi-asset market model**
- **Retirement income products**
- **Markov chain Monte Carlo methods**
- **Numerical results**
- **Conclusion**



Motivation



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Literature review – pricing techniques

- **Simulation based approaches:**
 - Monte Carlo
 - Least Squares Monte Carlo
 - Quasi Monte Carlo
- **Other numerical approaches:**
 - Partial differential equation
 - Tree based methods.
 - Stochastic control approach
 - Fourier Space Time stepping algorithm
 - Fourier cosine method
 - ...

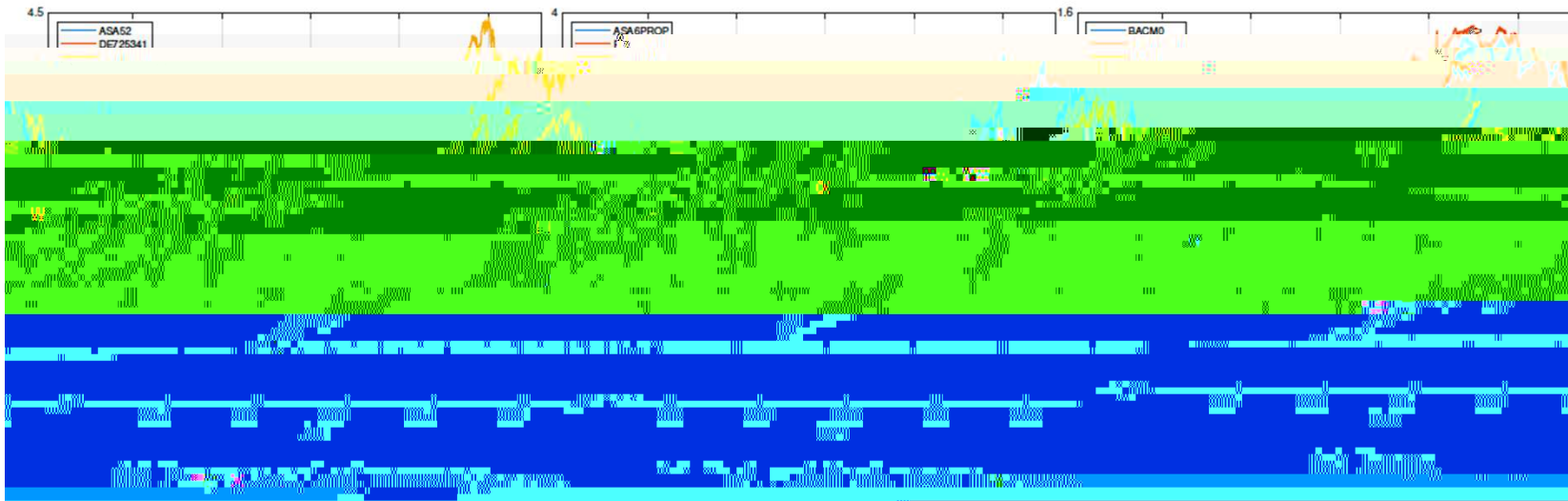


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Research questions

- **How to efficiently value retirement income products when the underlying investment fund consists of multiple asset classes?**
 - **Markov chain Monte Carlo (MCMC) algorithm**
- **How to devise a framework for retirement income product comparison**
 - **Longevity risk protection**
 - **Income volatility**
 - **Bequest**



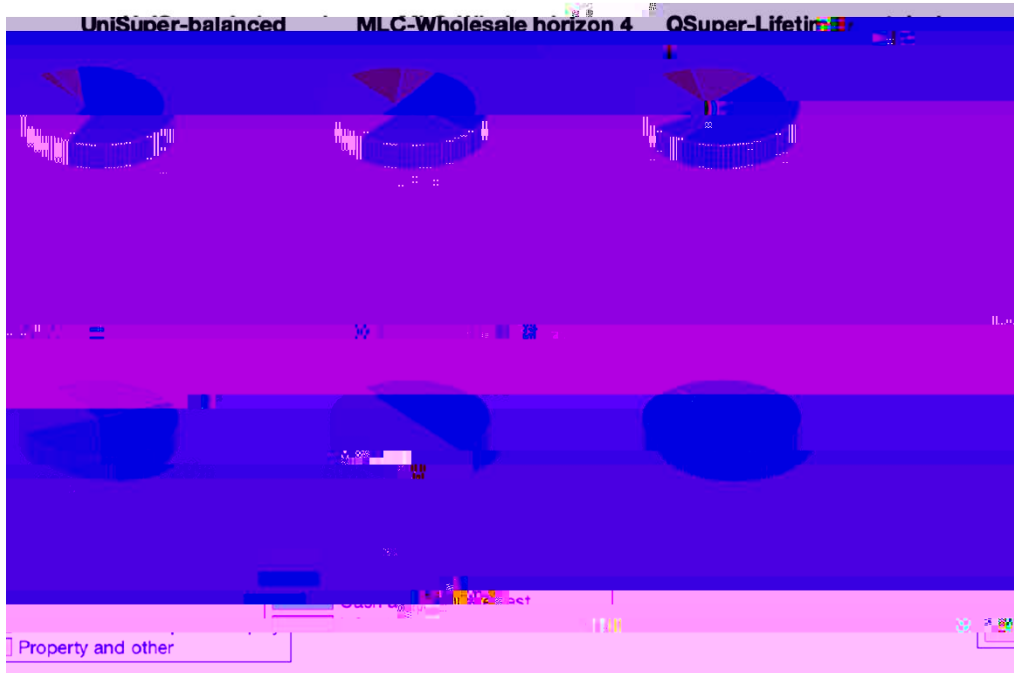


Market indices (APRA 2022)

Figure 1: Price observations of nine market indices² from February 2012 to March 2023. The prices at the beginning are normalised to one.

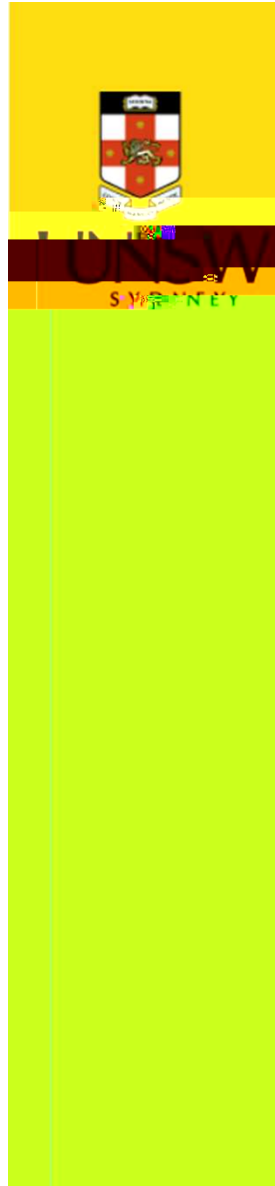
²The Australian Prudential Regulation Authority (APRA) choose nine indices to determine benchmark investment return for MySuper products (APRA, 2022).





The underlying fund invests in multiple asset classes

Figure 2 Some typical asset allocations of superannuation trustees in Australia
 (Source: providers' website).



The financial market

- **There are d assets in the market**
- **We assume that asset prices follow the geometric Brownian motion (GBM) process.**
- **Economic uncertainty: the regime-switching framework (Ignatieva et al, 2016).**
 - **Risk free interest rate.**
 - **Inflation rate.**
 - **Asset return and volatility.**



The financial market

- The demanded continuously compounded return of the assets



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The investment fund

- **Fund value before fee:**

$$F(t) = F(0) - \int_0^t c(s) ds$$

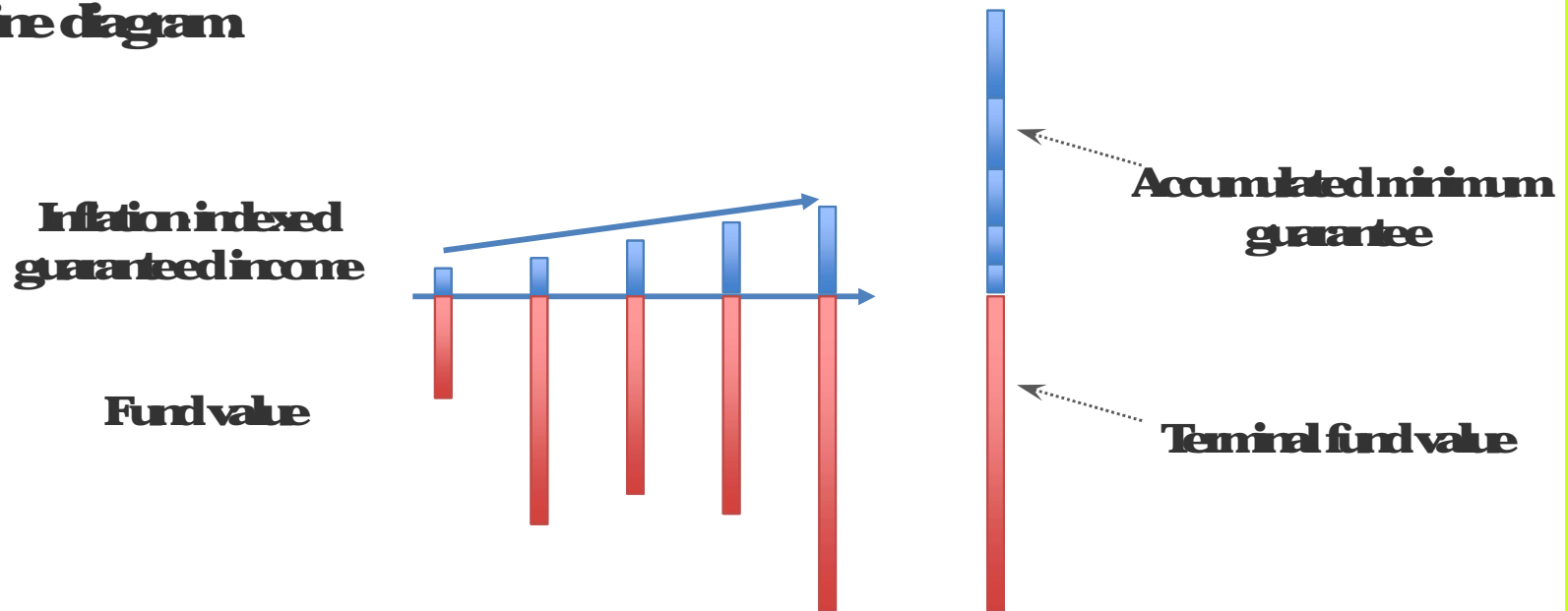
- **From the policyholder's perspective:**

$$\tilde{F}(t) := e^{-\zeta t} F(t).$$



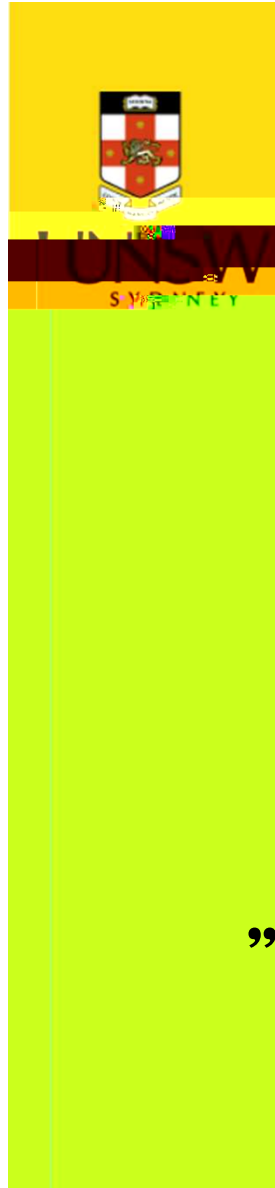
Guaranteed Minimum Income Benefit (GMIB)

- **Payoff = $\max(\text{terminal fund value}, \text{accumulated minimum guarantee})$.**
- **GMIB payment: Terminal fund value > accumulated minimum guarantee.**
- **Timeline diagram**



Growth lock-in feature for GMD

e a z

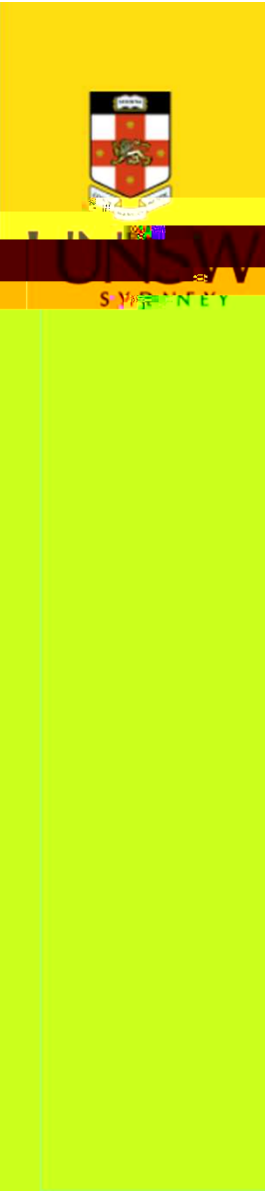




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GMWB in the market

Figure 4 An example of GMWB (MLC, 2022).



GMMB with spouse benefit option (MLC, 2022)

- **The spouse can continue making periodic withdrawals if the policyholder passes away**
- **Joint life model**



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Fees for variable annuities

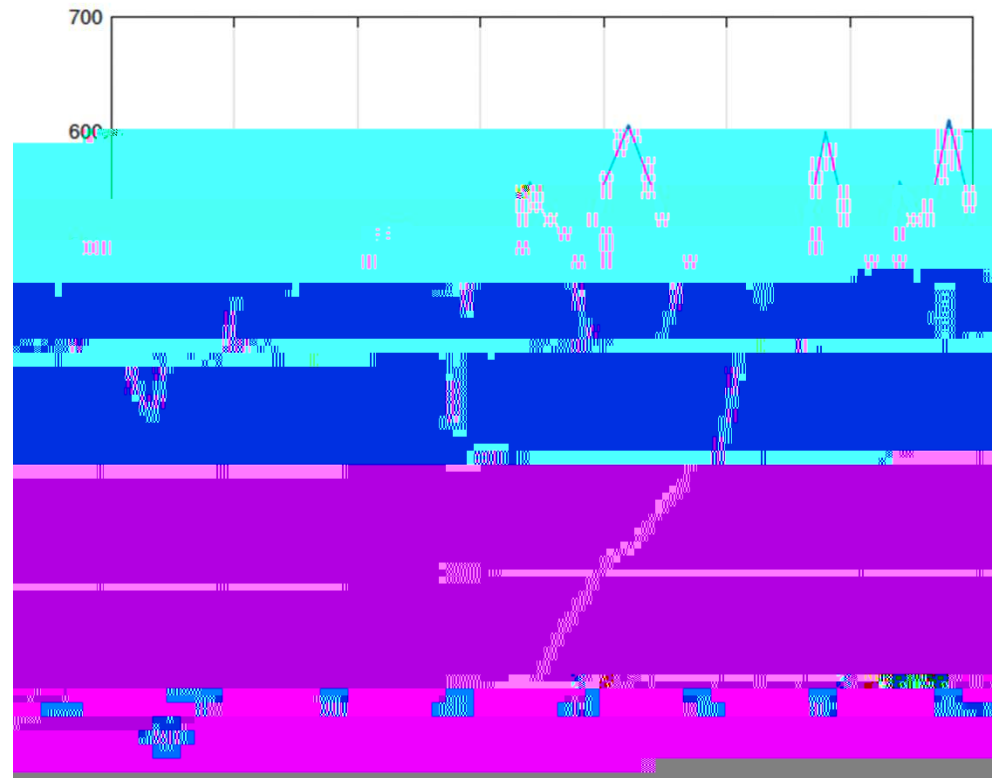
- Risk neutral pricing Expected payoff equals the initial investment
- We use the bisection method³ to solve for the fair management fee.

³Finding the root of a continuous function is frequently accomplished using the bisection approach. First, it locates an interval with the start and end of the interval having opposing function value signs. To identify the root, the approach then repeatedly bisects the interval with opposed function value signs at the start and end.



Account based pension

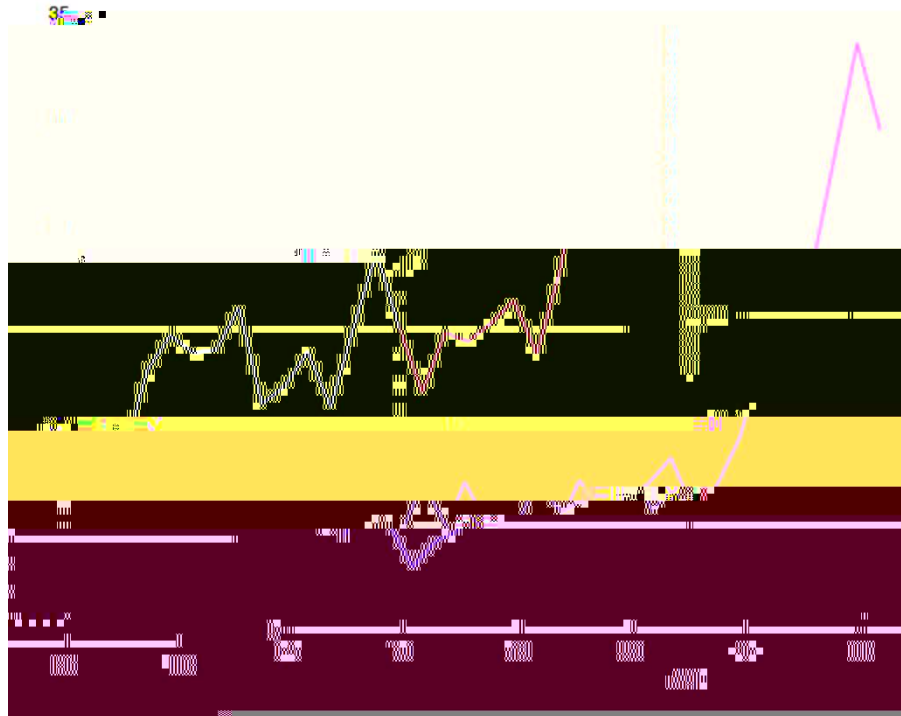
Figure 5 A simulated account based pension balance path that follows the minimum drawdown rate. The initial account value is \$100



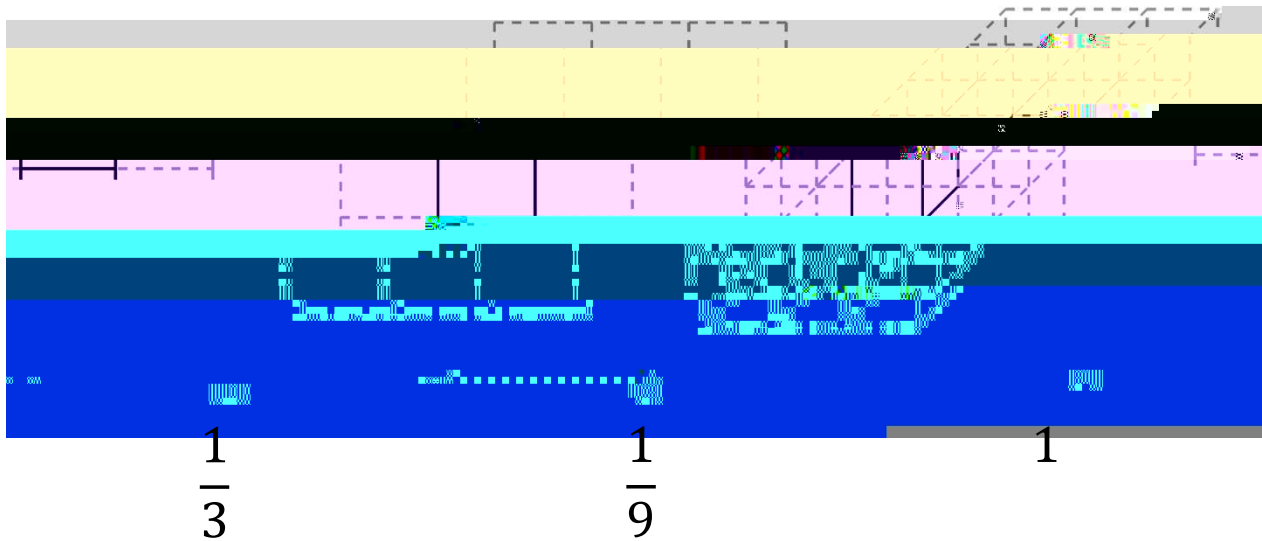
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GSA with spouse protection feature

Figure 6 A GSA with the spouse protection feature continues paying living benefits to the spouse if the member passes away (QSuper, 2022)

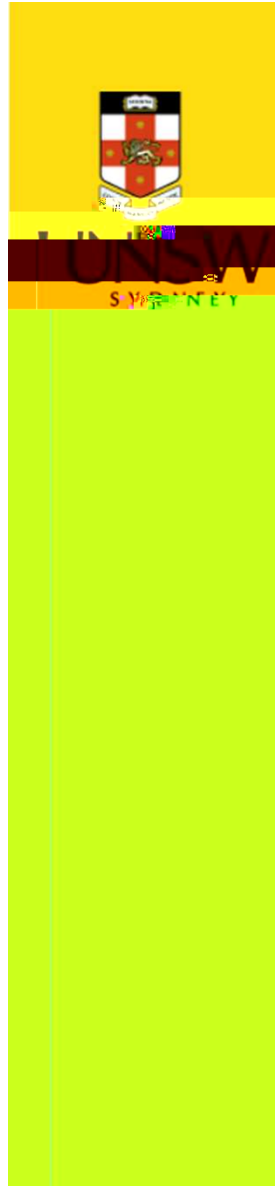


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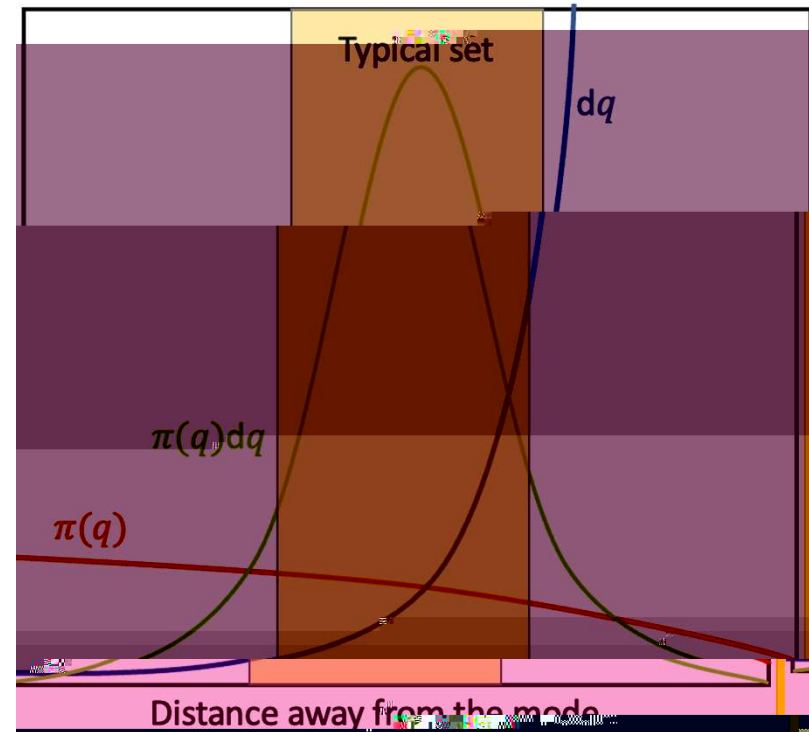
Volume of the parameter space

Figure 7: Consider a rectangular partitioning centered around a distinguished point, such as the mode.



Concentration of measure

- **Figure 8** In high dimensions, (\cdot) will concentrate around its mode, but the volume over which we integrate that density $\pi(q)$, is much larger away from the mode.



Assumption

- We consider a male policyholder aged 65, whose spouse is also aged 65
- The economy is in State 1 when risk free interest rate is less than 3% and in State 2 otherwise.
- We use the 9 indices⁵ chosen by Australian Prudential Regulation Authority (APRA) and the initial weights⁶ in the balanced investment option of UniSuper to construct the portfolio
- Our MCMC framework accommodates other asset classes whose distributional properties are known
- Human mortality follows the stochastic GoMa model (Qiao and Sherris, 2013).

⁵The Bloomberg ticker of the 9 indices we use are ASA52, DE725341, DN714533, ASA6PROP, RAHRSAH, FDCISAH, BACMD, LEGAIRAH, BAUEIL.

⁶The weights for Australian and international equity, cash and fixed interest, listed infrastructure, and listed property are 30%, 33%, 28%, 6%, and 3%, respectively. We assume the indices are of equal weight in each asset category.



Conclusion

In this research, we:

- **Devise an MCMC framework to efficiently value retirement income products in high dimensions.**
- **Conduct product comparison to reveal some insights.**
- **Extend the GSA design to allow for investment return adjustment**



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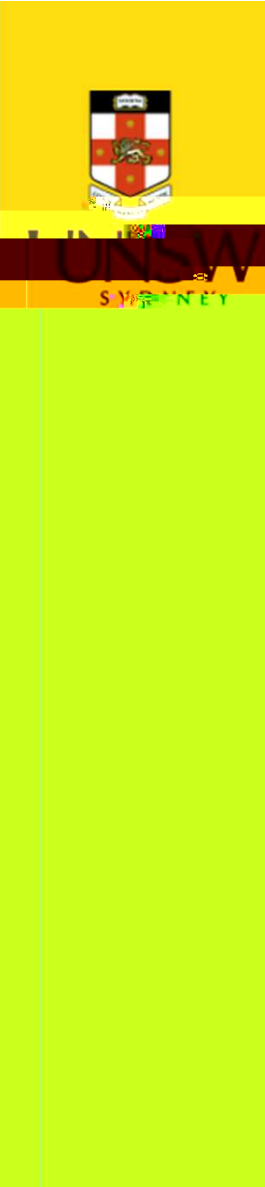
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THANK YOU!



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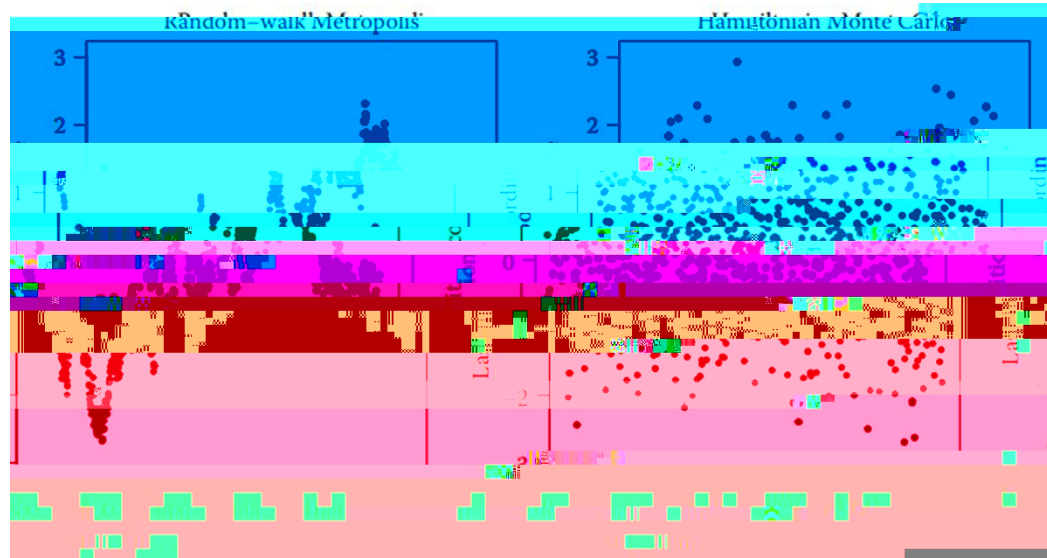
APPENDIX



RandomwalkMetropolis-Hastings (RWM) sampling

- **RandomwalkMetropolis-Hastings (RWM) sampling is a commonly used MCMC method in practice.**
- **RWM proposes a new state by random guess and does not work well in high dimensional simulation**
- **The Hamiltonian Monte Carlo (HMC) algorithm is a way to overcome this difficulty by proposing the new state according to the Hamiltonian dynamics (Neal, 2011).**





Drawbacks of the RWM method

Figure 18 An example in Neal (2011) where the authors simulate a 100 dimensional Gaussian distribution



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Differential geometry in the HMC algorithm

Figure 19 The node, gradient, and typical set are equivalent to the Earth, a gravitational field, and an orbit. Panel (a): if we only consider the gradient information and there is no momentum, the satellite will directly crash into the surface of the Earth. Panel (b): if the momentum is too small, the satellite will also crash into the surface of the Earth. Panel (c): if the momentum is too large, the satellite will escape the gravitational attraction. Panel (d): when we introduce the right amount of momentum, the satellite will move along the orbit.



HMC algorithm

- Origin from quantum physics (Alder and Wainwright, 1959).
- The Hamiltonian function (Neal, 2011):

where \mathbf{p} is an auxiliary d -dimensional momentum vector and



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HMC sampling

The main idea of HMC algorithm

- Start from the current state (\mathbf{r}, \mathbf{p}) on energy set
- Run the Hamiltonian dynamics for a duration
- Use the state $(\mathbf{r}', \mathbf{p}')$ at the end of the trajectory as the new state;
- Randomly draw a new momentum vector; the chain jumps to
- Repeat the previous steps.

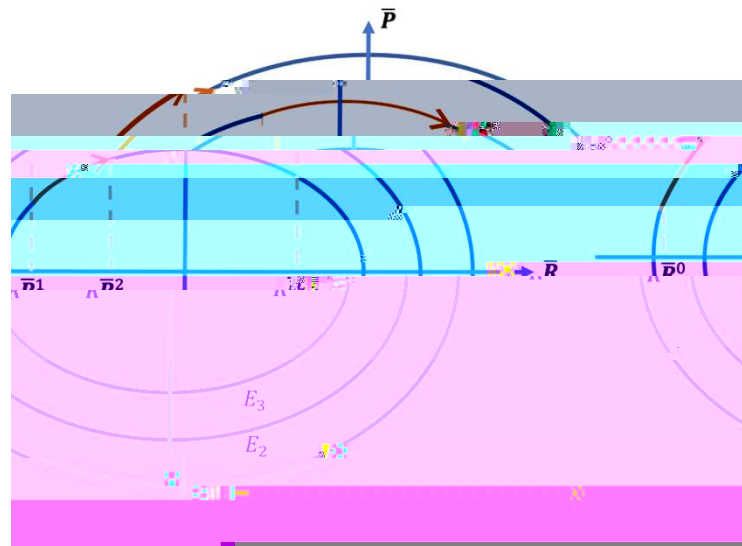


Figure 20 A sample path of the HMC algorithm, and are three energy sets. The horizontal axis is the target random variable, and the vertical axis is the auxiliary momentum vector we introduce.



Exact randomised Hamiltonian Monte Carlo (RHMC) algorithm

- Set M to



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Exact randomised Hamiltonian Monte Carlo (RHMC) algorithm

Proposition:

The change of variables

where

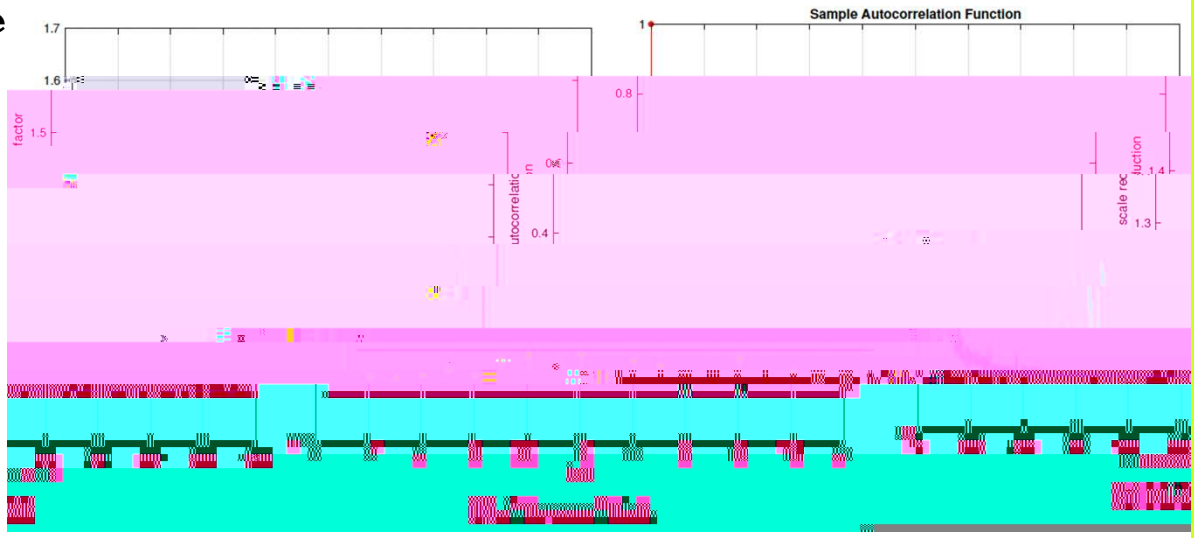


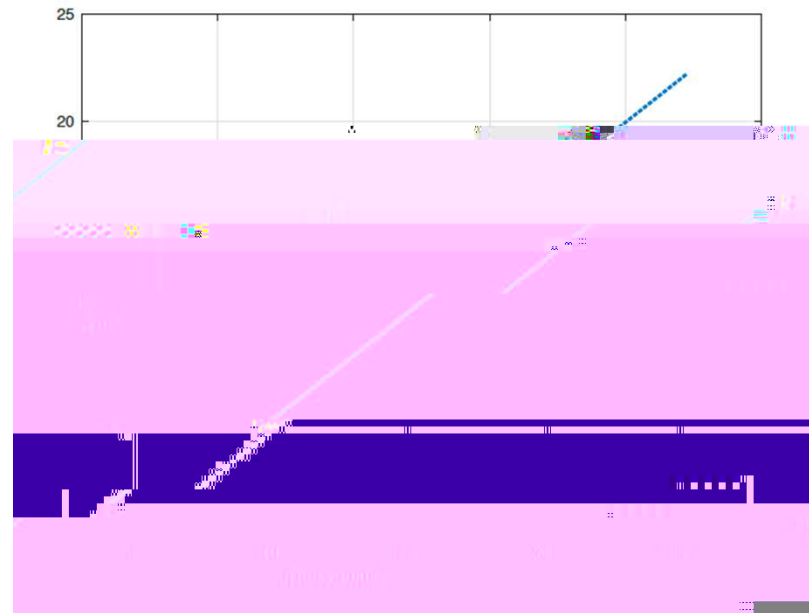
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Convergence test of the exact RHC algorithm

Figure 21: Convergence test of the exact randomised Hamiltonian Monte Carlo (RHC) algorithm
Panel (a) shows the estimated multivariate potential scale reduction factor (MPSRF) within 1,000 iterations. We simulate 5 Markov Chains with different initial starting points to estimate the MPSRF. The dimension is 9.
Panel (b) shows the sample autocorrelation of one stock return in one simulated Markov Chain



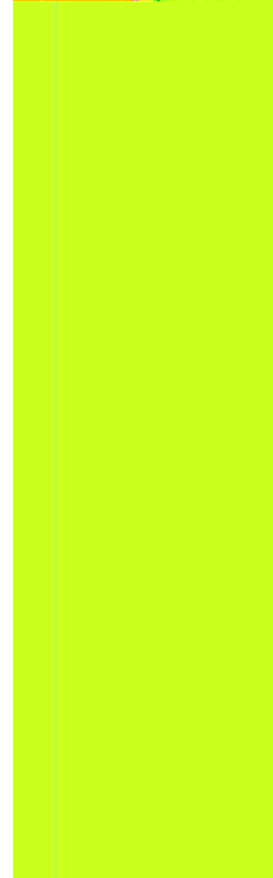


Convergence test of the exact RHM algorithm

Figure 22 True variances and sample variances of the 9 demeaned index returns. The sample size is $n = 100,000$.



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Mortality model

- We use the stochastic dynamic GoMa model in Qiao and Shen (2013) to model human mortality
- Let $\mu(x, t)$ be the mortality rate of a life aged x at time t , the mortality rate is given by:

$$\mu_x(t) = Y_1(t) + Y_2(t)c^x,$$

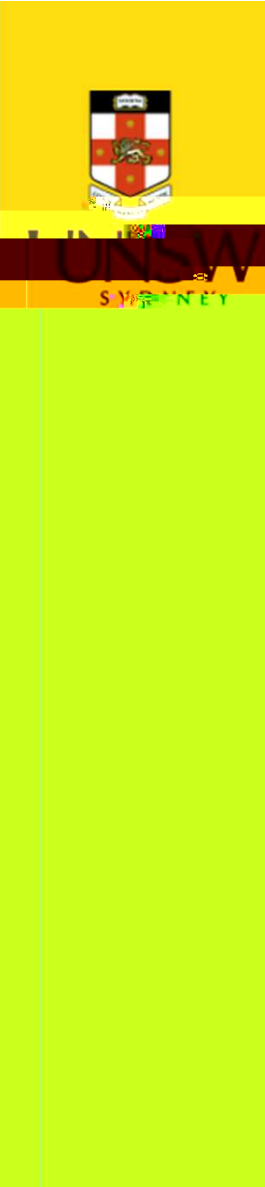
$$dY_1(t) = a_1 dt + \sigma_1 dZ_1^Q(t),$$

$$dZ_1^Q(t) dZ_2^Q(t) = 0 \text{ and } dZ_1^Q(t) dZ_1^Q(t) = dt,$$

where we assume $Z_1(t)$ and $Z_2(t)$ are independent of the financial market



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