A Cash Flow Approach to Expected Credit Loss Modelling

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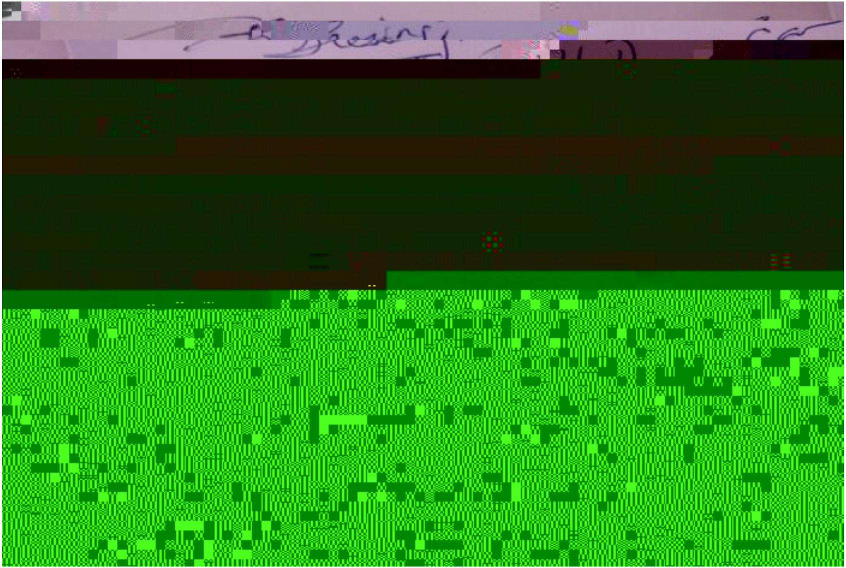
Collaboration between Industry and Academia



International Accounting Standards Board (IASB) requirements for IFRS9 and CECL

- Credit losses are the present value of all cash shortfalls over the expected life of the nancial instrument. A cash shortfall is the di erence between the cash ows that are due to an entity in accordance with the contract and the cash ows that the entity expects to receive. Both amount and timing of payments should be considered. [B5.5.28]; [B5.5.29{B5.5.35]

- Expected credit losses of a nancial instrument shall re ect an unbiased and probability weighted amount that is determined by evaluating a range of possible outcomes. [5.5.17, B5.5.41 { B5.5.43]

IASB ...Continued

- Expected credit losses shall re ect the time value of money. In particular, they shall be discounted to the reporting date using the e ective interest rate (EIR), except for purchased or originated credit-impaired nancial assets, in which case the credit-adjusted EIR is applied. [5.5.17, B5.5.44 { B5.5.48]

Standard ECL Calculation Framework

The commonly used Expected Credit Loss (ECL) calculation framework [KPMG 2017, etc.,] is given by

$$ECL = \sum_{j=1}^{m} PD_j \times LGD_j \times EAD_j \times D_j$$

where PD is the marginal probability of default, LGD is the loss given default, EAD is the exposure at default and D is a discount factor.

Thus, ECL is the discounted probability-weighted cash shortfalls discounted using the e ective interest rate

Towards an IASB Compliant Approach

The expected credit loss is computed as follows:

$$= V - (V_S + V_D)$$

where V is the PV of contractual cash ows under certainty, V_S is the PV of cash ows expected on default not occurring (survival) and V_D is the PV of recovery cash ows on default occurring.

$$V = \sum_{j=1}^{m} \frac{CF_j}{(1+b)^j}; \quad V_S = \sum_{j=1}^{m} !_j^0 \frac{CF_j}{(1+b)^j} \quad \text{and} \quad V_D = \sum_{j=1}^{m} !_j^1 \frac{R_j}{(1+b)^j}$$

$!_j^0$ be the probability equal to no default occurring up to time j, $!_j^1$ be the probability equal to default occurring between time $j-1$ and j and R_j is the recovery cash ow at time j R(and)TJ/F8

ECL Cash Flow Approach -1

We can write the expected recoveries cash ow as;

$$V_D = \sum_{j=1}^{m} \left({}^{1}_{j} (1 - {}_{j}) {}^{j} \sum_{i=j}^{m} \frac{CF_i}{\quad} \right.$$

ECL Cash Flow Approach -2

Given that:

$$V_D = \sum_{j=1}^{m} !_j^1 \; {}^{j} \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} - \sum_{j=1}^{m} !_j^1 \; {}_j \; {}^{j} \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i}$$

It is possible to re-arrange (1) such that:

$$= \sum_{j=1}^{m} !_j^1 \; {}_j \; {}^{j} \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} + \sum_{j=1}^{m} (1 - !_j^0) \frac{CF_j}{(1+b)^j} - \sum_{j=1}^{m} !_j^1 \; {}^{j} \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i} \tag{2}$$

as a correction term

The correction term, , can be written as

$$= \sum_{j=1}^{m} \frac{CF_j}{(1+b)^j} - \sum_{j=1}^{m} \frac{CF_j}{(1+r)^j} - (1 - \quad) \sum_{j=1}^{m} q_j \frac{CF_j}{(1+b)^j}$$

$$+ \; (1 - \quad) \sum_{j=1}^{m} q_j \; {}_j \sum_{i=j}^{m} \frac{CF_i}{(1+r)^i}$$

In the one period case ($m = 1$), $= 0$. When the e ective interest rate (or reference rate) is equal to the contractual rate then $= 0$, thus the commonly used ECL formula is accurate.

It can also be shown that when $b > r$, as is commonly the case, that < 0. Therefore in most applications the commonly used ECL formula is a conservative estimate of ECL.

Example A: bullet loan

A xed rate bullet bond with interest rate paid at maturity has cash ows given by $CF_i = 0$ for $i < m$ and $CF_m = (1+r)^m N$. The expected credit loss (ECL) is given by

$$= N \sum_{j=1}^{m} (q_{j-1} - q_j) \;_{j}\; ^{j} +$$

where

$$= N \left(- \quad + \quad ^{m} - (1 - \quad) q_m \;^{m} + (1 - \quad) \sum_{j=1}^{m} q_j \;^{j} \right)$$

Example - A bond loan

A xed rate bullet loan with period interest payments is de ned by cash ows given by $CF_i = rN$ for $i < m$ and $CF_m = (1 + r)N$

Example C - amortizing loan

A credit facility o ered to a borrower for which there is a payment of both principal and interest. r, are made with each payment. The repayment amount is $P = r(1+r)^m N=((1+r)^m - 1)$, the ECL is;

$$= \frac{N}{(1+r)^m - 1}\sum_{j=1}^{m}(q_{j\ \ 1} - q_j)\ _j\ ^j\ \left((1+r)^{m\ \ j+1} - 1\right) +$$

where

$$= N \left[\frac{r}{b}\left(\frac{(1+b)^m - 1}{(1+r)^m - 1}\ ^m - \ \right) - (1 - \ )\frac{r(1+r)^m}{(1+r)^m - 1}\sum_{j=1}^{m}\frac{q_j}{(1+b)^j} + (1 - \ )\frac{1}{(1+r)^m - 1}\sum_{j=1}^{m} q_j\ ^j\ \left((1+r)^{m\ \ j+1} - 1\right)\right]$$

Replication Theory and Expected Credit Losses

The ECL for an amortizing loan with periodic repayments, P, can be expressed as ECLs of a xed paying loan and a bullet loan with notionals N^b and N^e, respectively. where $N^b = \quad N$, $N^e = (1 - \quad)N$ and

$= (1 + r)^m = ((1 + r)^m - 1)$

The ECL for an amortizing loan is thus gTJ/]TJ/F53 10.9091 TThe

Numerical Examples

Product	ECL	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Bullet	ECL(1 yr)	24.72	24.23	23.52	22.39
Loan	ECL(5 yrs)	117.62	106.52	91.84	71.78
	ECL(10 yrs)	221.40	181.60	134.98	82.46
Coupon	ECL(1 yr)	23.63	23.17	22.51	21.44
Bond	ECL(5 yrs)	93.51	85.42	74.66	59.81
	ECL(10 yrs)	143.66	122.06	96.14	65.66
Amortising	ECL(1 yr)	13.24	13.06	12.79	12.37
Loan	ECL(5 yrs)	56.16	52.72	48.03	41.26
	ECL(10 yrs)	98.12	87.18	73.39	55.82

Table 1: Expected Credit Loss for three loan products with, 1,000 notional, 5% default probability, a contractual rate of 10% under the four discount (e ective interest) rate scenarios, 10%; 12%; 15% and 20%; respectively.

Conclusion

- We show that the formula most commonly applied in the literature for calculating lifetime expected credit loss is inconsistent with measuring expected loss based on expected discounted cash ows.
- Valuation framework presented is exible and can be used for loans with oating interest rates using forward rates
- In this framework we can easily incorporate prepayment options
- We can use replication theory to compute Expected Credit Losses for loans with complex cash ows