

SOSS1000

Dc`]WnUbX'GcWJYhm

HYfa 'C bY''##'&\$&%

7ci fgY Cj Yfj]Yk

GHJZZ:7cbHJWhi8YHJ]`g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
8]UbU'DYfWXY	Ţ,	-	A cfj Yb [.] 6fck bž C ZZJWY [.] % ,	7 cbHJWhj]U HYUa g

@YWhi fYfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D∖cbY
5`]gcb'K UbbUb	U"k UbbUb4 i bgk "YXi "Ui	D`YUgY'Ya U]`'hc UffUb[Y'Ub Uddc]bha Ybh'		

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`YI d`cfYg`h\Y`bUh fY`cZh\Y`Udd`]YX`gcWJU``gWJYbWYg`UbX`dfcj]XYg`]bZcfa UhJcb`UVci h\ck `gcWJU` gWJYbWY`WUb`VY`Udd`]YX`hc`fYU``k cf`X`dfcV`Ya g"`Mci `k]```VY`]bHfcXi WYX`hc`]XYUg`UVci hih\Y`fc`Y`UbX`i gY`cZ Yj]XYbWY`]b`XYWJg]cb!a U_]b[`UbX`dc`]Whla U_]b[`Uh`cWU`ž'bUhJcbU``UbX`]bHYfbUhJcbU```Yj Y`g"`K Y`U`gc]bj YghJ[UhY`\ck 'dc`]Whi]g`XYj Y`cdYXž'W\Ub[YXž]a d`Ya YbHYX`UbX`Yj U`i UhYX`]b`Udd`]YX`gcWJU``gWJYbWY Xca U]bg`gi W\`Ug`gcWJU``dc`]Whž'di V`]Wdc`]Whž'Wf]a]bU``^ ghJWY`UbX`gcWJU``k cf_"`K Y`k]```U`gc`X]gW' gg`h\Y dfUWJ\WJ`Zdc`]hJWU`UbX`Yh\]WU``dfcV`Ya g`YbWci bhYfYX`]b`h\Y`Wca a i b]\WJhJcb`cZfYgYUfW\`UbX`YI dYff]gY`hc]bZcfa 'dc`]WhiUbX`gcWJU``W\UbX`YI dYff]gY`hc

 $\label{thm:conditional} $$H\Y'Wci fgY']blfcXi WYg'gh XYbhg'hc'_Ym]XYUgžXYVUhYg'UbX']ggi Yg'k]h\]b'h\Y'gc\]U''gV]YbWYg']b fY`Uh]cbg\]d'hc'dc`]WmUbX'dfUW]WY'''K Y'XYj Y`cd'_YmghfUhY[]Yg'UbX'g_]``g'h\Uhgh XYbhg'Wb'Udd`mhc dUfh]W'''Uf'UfYUg'cZYa d`cma Ybh]b'h\Y'_YmUfYUg'cZdi V']WUbX'gc\]U''dc`]\Mz'gc\]U''k cf_'UbX'\Mf]a]bU` $$ gh]WY'''5 gYf]Yg'cZfYU'!k cf'X'dc`]\mwUgY'gh X]Yg'\][\`][\\h]b['W ffYbhfYgYUfW\'k]``'VY'i gYX'hc']`i ghfUhY h\Y'\cbY\MfCbg'VYhk YYb'dc`]\mz'dfU\Mf\W' UbX'\W ffYbhXYVUhYg''$

7 ci fgY @YUfb]b[Ci hMca Yg

- %" 5dd`m`Wc[b]hjj Y'g_]``g`hc`fYj]Yk ž'UbU`mgYž'Wcbgc`]XUhY`UbX`gmbh\Yg]gY`_bck`YX[Y'dYfh]bYbh'hc Udd`]YX`gcW]U``gW]YbWY`X]gW]d`]bYg`gi W\`Ug`GcW]U``F YgYUfW\`UbX`Dc`]Whž'7f]a]bc`c[m`UbX`GcW]U` K cf
- &" 8 Ya cbgffUhY 'Ub 'UV]`]hmhc 'UXUdhiUbX'i gY 'X]gW]d`]bUfmi_bck `YX[Y 'UbX'fYgYUfW\ 'g_]``g']b 'Udd`]YX gcW]U``gW]YbWY 'WcbhYI hg
- ' " I gY`]b]h]Uh]j Y`UbX`^ X[Ya Ybh]b`Udd`]YX`gW\c`Ufg\]d`]b`GcW]U``F YgYUfW\`UbX`Dc`]Whž`7 f]a]bc`c[m UbX`GcW]U``K cf_
- ("=bhYfdfYhUbX'Wcaaib]WUhY'fYgYUfW\Z]bX]b[g'hc'X]ZZYfYbhUiX]YbWYg'Udd`]WUV`Y'hc'h\Y'Udd`]YX gcW]U`gW]YbWYg

HYUWX]b[GlfUhY[]Yg

K Y'gYY'I BGK 'ghi XYbh``YUfbYfg`Ug'dUfh]W]dUbhg`]b`h\Y'dfcXi Wf]cb`cZ_bck``YX[Y'UbX'kY'di fgi Y'h\Y]a d`Ya YbhUh]cb`cZU

W ff]W `i a 'fYgdcbg]j Y'hc'YUWX 'ghi XYbhfgi']bX]j]Xi U``bYYXg"'H\Y'XYg][b'cZh\]g'Wci fgY']g'a chjj UhYX'Vm'U XYg]fY'hc'ZcghYf'h\Y'g_]``g'cZ]bXYdYbXYbh'`YUfb]b['UbX'Wf]h]WU'`h\]b_]b["

H\YfY`UfY`t\fYY`a U]b`hmdYg`cZghfi Whi fYX``YUfb]b[`UWhjj]hmi]b`t\]g`Wci fgYz`gi dd`Ya Ybh]b[`]bXYdYbXYbh qhi XmffYUX]b[z`bchY!hU_]b[z`fYqYUfW\Ł

%" @97HI F 9G

H\]g`Wci fgY`Wcbg]ghg`cZk YY_`m`Uf[Y![fci d``YWi fYgžk \]W\`a Um`]bWi XY`h\Y`dfYgYbhUh]cb`cZj]XYc`W]dgž [fci d

UWIj]hjYg`UbX`[fci d`X]gWi gg]cbg"'H\Y``YWi fYg`UfY`XYg][bYX`hc`VY`dUfhjWjdUhcfmUbX`Yb[U[]b[gi dd`Ya Ybhjb[``]bXYdYbXYbhighi Xm]b`h\Y`Wci fgY`h\UhYbWci fU[Yg`WcgY`Yb[U[Ya Ybhik]h\`k YY_`m fYUX]b[g"

&""HI HCF =5@G

Hi hcf]U`g`U`ck 'ghi XYbhg`hc 'X]gWi gg`h\YcfYh]WU``UddfcUW\Ygž`Ug_'ei Ygh]cbgž'Z fh\Yf`]bhYffc[UhY`WcbW'dhg UbX`]ggi Yg`Uf]g]b['Zfca 'h\Y]f`]bXYdYbXYbhghi Xm" Hi hcf]U`g`U`gc`dfcj]XY`ghi XYbhg`k]h\`h\Y`cddcfhi b]hmhc

 $Yb[U[Y']b'dfcXiWfjY'X]gWigg]cb'k]h'N'h'NY]f'dYYfg'UbX'hc'dUffjWjdUhY']b'U'fUb[Y'cZghfiWifYX'YUfb]b[UWfj]h'N'gz']bWiX]b['[fcid#dU]f#]bX]j]XiU''UbU'mfjWU''kcf_z'zCfaU''UbX']bZcfaU''XYVUhYgz'ei]nnYg'UbX'a]b]!dfcYWfq'''$

'"-\$89D9B89BH~@95FB\$; '57H\$\#\-9G

±DXYdYbXYbhiYUfb]b['UWfj]hjYg'a i ghVY'Wta d'YhYX'YUWk'k YY_"'H\Y'`YUfb]b['UWfj]hjYg'UfY'XYg][bYX'ht ZcghYf']bXYdYbXYbhghi Xm'UbX'ht 'U`ck 'Zcf'Yb[U[Ya Ybhk]h\ 'X]ZZYfYbhiHug_g'h\UhWUbbchVY'i bXYfhU_Yb']b'U hi htf]U`Ybj]fcba Ybhz'gi Wk'Ug'WtbXi Wfjb['cb`]bY'fYgYUfWk'cf'k UhWk]b['U'Z]'a "'Ghi XYbhg'k]``'U'gc'i bXYfhU_Y ZJY`Xk cf_'VmcVgYfj]b['UbX'fYdcfh]b['cb'fYU'!']ZY'dc`]Wma U_]b["'5 Wfj]hjYg'UfY'k]``'VY'YI d`U]bYX'UbX'k]`` a cghmVY'ZJWj]hUhYX'Vmh\Y'I b]j Yfg]hm@YUfb]b['A UbU[Ya YbhGmghYa "'

5ggYgga Ybh

D`YUgY`gYY`AccX`Y`Zcf`XYHJ]`YX`UggYggaYbh]bZcfaUh]cbž`]bWiX]b[`aUf_]b[`Wf]hYf]U"`

5ggYgga Ybh'HUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
A]bcf'9ggUm	' \$1	%&#\$' #&\$&%'\$).\$\$'DA</td><td>%<u>*</u>''</td></tr><tr><td>:]Y`Xk cf_`F Ydcfh</td><td>&\$I</td><td>\$-#\$(#&\$&%`\$).\$\$`DA</td><td>&ž¨'</td></tr><tr><td>A U⁄cf'9ggUm</td><td>) \$1</td><td>'\$#\$(#&\$&%`\$).\$\$`DA</td><td>&ž˙(</td></tr></tbody></table>	

5ggYgga Ybh'8YhJ]`g

5qqYqqa Ybh% A]bcf 9qqUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h. %28\$\$ k cfXg

8YHJ]`g.

 $\label{lem:continuity} Gh XYbhg gi Va]hU g \cfhYggUmcZ% $$ k cfXg cb hY W bhYbhcZhY ZfghgYhcZYWh fYg" H]g YggUm U ck g gh XYbhg hc Yl d cfY hY]ggi Yg]b [fYUhYf XYhU] "$

Ghi XYbhg`k]```VY`dfcj]XYX`k]h\`kf]hhYb`ZYYXVUW_'U`cb[g]XY`U`a Uf_]b[`fi Vf]Wcb`h\Y]f`gi Va]hhYX`k cf_" 5qqYqqa Ybhq`k]```VY`a Uf_YX`UbX`fYhi fbYX`k]h\]b`%\$`k cf_]b[`XUmq"

5XX]h]cbU`XYhJ]`g.

9ggUmei Yghjcbg k]``VY dcghYX cb A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$

5ggYgga Ybh&:]Y Xk cf_ F Ydcfh

@Yb[h\.**\$\$*kcfXq

8YHJ]`g.

Ghi XYbhgʻgi Va]hUʻ*\$\$ik cfXʻZJYʻXk cf_ʻfYdcfhʻiH\]gʻ]gʻXcbYʻUZYfik UhW\]b[ʻUʻ`cWUʻ[cj Yfba Ybha YYh]b[fY]h\Yfʻ]bʻdYfgcbʻcfʻcb`]bYłžʻUbXʻUbUʻmg]b[ʻh\YʻdfcWYX]b[gʻ'H\YʻfYdcfhik]```Uʻ`ck ʻghi XYbhgʻhcʻUdd`mih\Yh\Ycfm`YUfbh`]bʻh\YʻWci fgYʻhcʻh\YʻfYUʻ`k cf`XʻcZdc`]Wna U_]b[ʻ'Ghi XYbhgʻk]```VYʻdfcj]XYX`k]h\k f]hhYb ZYXXVUW_ʻUʻcb[g]XYʻUʻa Uf_]b[ʻfi Vf]Wcbʻh\Y]fʻgi Va]hhYX`k cf_ʻ'5ggYgga Ybhgʻk]```VYʻa Uf_YX`UbXʻfYhi fbYX k jh\]bʻ%\$ik cf_]b[ʻXUmgʻ'

5XX]h]cbU`'XYhU]`g.

 $+ bghfi Whjcbg'Zcf'h Y'ZjY'Xk cf_'hUg_k]``VY'dfcj]XYX']b'WUgg'UbX'cb'A ccX`Y''$

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh]g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb g]a]'Uf]hmfYdcfhg"

5ggYgga Ybh' . A U'cf 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&ž&\$\$

8 YHJ] g.

Ghi XYbhigi Va]hU``cb['YggUmcZ&ž&\$\$'k cfXg"'9ggUmei Ygh]cbg'k]```VY`a UXY`Uj U]`UV`Y`]b`WUgg'UbX'cb`]bY" H\]g'YggUmU``ck g'ghi XYbhg'hc'Yl d`cfY`_Ym]ggi Yg']b`[fYUhYf'XYhU]`"

Gh XYbhg k]``VY`dfcj]XYX`k]h\ k f]hhYb ZYYXVUW_'U`cb[g]XY`U`a Uf_]b[`fi Vf]Wcb`h\Y]f`gi Va]hhYX`k cf_"

5ggYgga Ybhg'k]``'VY'a Uf_YX'UbX'fYhi fbYX'k]h\]b'%\$'k cf_]b['XUmg"

5XX]h]cbU`XYhU]`g.

9ggUmei Yghjcbg k]``VY`Uj Uj`UV`Y`cb`A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5 hm/b XUb WY F Yei]fYa Ybhg

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YHJV`Y

8UH/	HmdY	7 cbhYbh
K YY_'%'%)': YVfi Ufm!'% : YVfi Ufm	@YWhi fY	

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

⊋mci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\'Hi fb]hJbž'd`YUgY hY`Yd\cbY'9| hYfbU`'Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hYhgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh\ci fg'UfY', .\$\$Ua '-',\$\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ'''⊋mci UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi `hk]h\'Hi fb]hJb'mci 'a UmUdd`mZcf'Ub'Y! hYbg]cbž'Vi hmci a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU`'Gi ddcfhfU`cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a YbhgŁ'hc

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfglUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbhUhi BGK
- VY 'Uk UfY 'cZd`U[]Uf]ga ž Wcdmf][\hzi BGK 'Ghi XYbhi 7 cXY 'cZ 7 cbXi WhUbX '5 WW dhUV 'Y 'I gY 'cZ I BGK '=7 H'F Ygci fW g'Dc`]Wh
- VY 'Uk UfY 'cZh\Y'ghUbXUfXg'cZVY\Uj]ci f'Yl dYWhYX'cZYj YfmcbY]b 'h\Y'l BGK 'Wca a i b]h

5 WUXYa]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua]bUh]cb dfcWYXi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]miUbX'X]qUV]`]m/
- UbX'ch\Yf'YggYbhJU`UWUXYa]W]bZcfa UhJcbz'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

.

⇒a U[Y'7fYX]h

DUf]Ua YbhJfmlf]Ub[`Yž7 UbVYffU" D\chc Vm8 DYfWXY

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb : `W ghcX]Ubg : cZh, Y ``UbXg : cb : k \]WX : I B G K ? Ybg]b[hcb : WUa di g :]g ``cWUhYX''$