7ci fgY^{*}Cj Yfj]Yk

[']GhJZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
7`Ui X]U'HUnfY]hYf	WMUnfY]hYf4 i bgk "YXi "Ui	Hi YgXUm%&da	A 6 ° %* &	-',)'*-%

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWkcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

 $BchY. H\ [g']g'U'\% I C7'WcifgY''Mcik]``Ybfc``]b'* I C7']b'YUWX'cZhk c'giWW'gg]j Y'hYfa g'k]h\ 'U'fYgi`h fYdcfhYX'Vm'U'g]b[`Y'[fUXY'Uh'h\Y'YbX'cZh\Y'gYWcbX'* I C7'WcifgY''$

H\]g']g'U'WcfY'Wci fgY'Zcf'<cbci fg'WUbX]XUHYg']b'h\Y'GW\cc``cZGcW]U``GW]YbWrg'UbX'h\Y'a U]b'ZcW'g'k]```VY h\Y'ei Ygh]cb'cZa Yh\cXc`c[m''9UW\gYa]bUf'k]````cc_'UhX]ZZYfYbh'UddfcUW\Yg'hc'fYgYUfW\'UWfcgg'h\Y'gi V!X]gW]d']bYg'UbX'k]```YI d'cfY'k\Uhk Y'a][\h\W\``h\Y''gc'k\UhrcZU'dUfh]W``Uf'a Yh\cXc`c[mg'UbU`mf]WU``Yj YfU[Y''K\mXc'h\Y'GcW]U``GW]YbWrg'VfYU_']bhc'a mf]UXž'Wca dYh]b['a Yh\cXc`c[]Yg3'7 Ub'k Y'XYgWf]VY ci f'a Yh\cXc`c[]Yg'Ug'gW]Ybh]Z]Wz'UbX']Zbchz'h\Yb'\ck 'Xc'k Y'VYghgYW'fY'U'bch]cb'cZUWW'fUWmUbX i gYZ\`bYgg3'<ck 'XcYg'ci f'i bXYfghUbX]b['cZYj]XYbW''UWh'U`mW\Ub[Y'ci f'fYgYUfW\'a Yh\cXg'Ug'k Y``'Ug h\Y'ci hWca Yg'UbX'WcbWi g]cbg'h\Uhk Y'a][\hfYUW\3

7 ci fgY @YUfb]b[Ci hWca Yg

```
%" 5dd`m`Wf]hJWU``th`]b_]b[ 'UbX'UbU`mf]Wg_]``g
&" 9ZYWf] Y`mXYd`cmU'dUffjWi`Uf`th`Ycfmcf`a Yth`cXc`c[ ]WU``UddfcUW\
' " 8Yg][ b`UbX'gi ghU]b`Ub'Uf[ i a Ybhz'cf'dc]bhcZj ]Yk
( "
```

 $\label{eq:linear_model} $$Mci 'a i ghigi Va]$hmci f'X]gW gg]cb'dUdYf'hc'Hi fb+hb']b'A ccX`Y'Vm'$$Ua 'H\i fgXUm'&*h\'A UfW\'''=k]``bch'UWWdhigi Va]$gg]cbg'VmYa U]`'''$

5XX]h]cbU`'XYhU]`g.

 $H\Y'h\] fX'UbX'Z\] bU``UggYgga Ybh'HJg_'Zcf'h\] g'Wci fgY']g'h\Y'fYgYUfW\'dUdYf''' + h\] g'dUdYfž'mci 'a i ghUXXfYgg'h\Y'Zc``ck]b['h\fYY'ei Ygh]cbgž'k \]W\'UfY'h\Y'gUa Y'h\fYY'hcd]Wg'h\Uhh\Y'[i YghdfYgYbhYfg'\Uj YUXXfYggYX'YUW\'k YY_'h\fci [\ci hh\Y'gYa YghYf.$

- %" K \UhiX]gW]d`]bUfmUbX`gW\c`Uf`miffUX]f]cbg`#dYfgdYWf]j Yg`#gYbg]V]`]h]Yg`Xc`mci `Vf]b['hc'h\Y fYgYUfW\'h\Uhimci 'k]``Xc'h\]g`mYUf3`
- $\label{eq:continuous} $$ ''' < ck' Xc'h\YgY''YUX'hc'dUff]W'`Uf'k Umg'cZi bXYfgHJbX]b[''UbX'fYgYUfW\]b[''h\Y'gcW]U'`k cfX3'K \UhUfY'h\Y'ghfYb[h\g''UbX'`]a]HJh]cbg'cZmci f'X]gW]d']bUfm#gW\c`Uf'mdYfgdYWf]j Y3'$

 $\label{thm:condition} $$ Mci 'g\ci 'X'YI d`cfY'h\YgY'ei Ygh]cbg'UbX'fYZYWhi dcb'h\Ya 'VYZcfY'k f]h]b['i d'U'h\ci [\hZ ``UbX'dYfgcbU` mYhgWkc`UfmdUdYf''9UW 'gYWfcb'g\ci 'X'XYj Y`cd'Ub'Uf[i a Ybhcf'gYhcZUf[i a Ybhg'UbX'h\Y'dUdYf'Ug'U k \c`Y'g\ci 'X'VY'Vch\'Wc\YfYbhUbX'Wc\Yg]j Y'''K Y'k]```gdYbX'h]a Y']b'WUgg'X]gW gg]b['\ck 'hc'UddfcUW h\]g'UggYgga Ybhz'UbX'Z fh\Yf'fYgci fWg'UfY'Uj U]`UV'Y'cb'A ccX`Y'''$

 $\label{eq:higher} \begin{tabular}{ll} High \$

5 https://bxubwww.Fyei]fya Ybhg

D`YUgY`bchY`h\Uh`YWhifY`fYWcfX]b[g`UfY`bchïUj U]`UV`Y`Zcf`h\]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifU[YX`hcUhhYbX`U```WUggYg`UbX`WcbhUWhh\Y`7cifgY`5ih\cf]hmhc`aU_Y`U`hYfbUh]jY`UffUb[YaYbhg`Zcf`WUggYga]a]ggYX"

7ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YHUV'Y

Fygci fWyg

DfYgWf]VYX`F Ygci fWYg`

 $\label{eq:K-YY_mapping} K YY_`m`]bglfi Wf]cbg`UbX`fYgci fWfg`k]```VY`Uj U]`UV`Y`cb`A ccX`Y''$

FYWca a YbXYX FYgci fWyg

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWUb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU``mWcdm]b['Zfca `U`gci fW'k]h\ci hUW_bck `YX[Ya Ybh'

 $I BGK `[fci dg'd`U[]Uf]ga `]bhc'h\Y'Zc``ck]b[`WUhY[cf]Yg.$

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/