

ARTS3756

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8YHJ]`g. `Ghi XYbhg`]XYbh]Zm`UbX`UbU`mgY`W ffYbhXYVUhYg`]b`U`WXcgYb`UWUXYa]Wcf`dc`]WmhYl hz`XfUk]b[`]bUXX]h]cbU```]hYfUhi fY`UbX`Yl Ua d`Yg"'&\$\$\$`k cfXgGhi XYbhg`k]```fYWY]j Y`k f]hYb`ZYYXVUW_'UbX`U`bi a Yf]WU` [fUXY`k]h\]b`&`k YY_g`cZgi Va]gg]cb"'H\Y`ZYYXVUW_'g\YYhk]```VY`Uj U]`UV`Y`Zfca 'h\Y`VY[]bb]b[`cZgYa YghYf`gc'h\Unghi XYbhg`WUb`k cf_'hck UfXg`gdYW]ZYX`ghUbXUfXg"'`

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8 YHJ]`g. `Ghi XYbhg`]XYbhJZm`UbX`UbU`mgY`W ffYbhXYVUhYg`]b`U`fYWYbhYj YbhX]gWi ggYX`]b`WUgg`cf`gci f\WX h\fci [\`a YX]Už`XfUk]b[`]b`UXX]hJcbU```]hYfUhi fY`UbX`YI Ua d`Yg"`&\$\$\$`k cfXgGhi XYbhg`k]```fY\W]j Y`k f]hHYb ZYYXVUW_`UbX`U`bi a Yf]\WJ``[fUXY`k]h\]b`&`k YY_g`cZgi Va]gg]cb"`H\Y`ZYYXVUW_`g\YYhk]```VY`Uj U]`UV`Y`Zfca h\Y`VY[]bb]b[`cZgYa YghYf`gc`h\Uhghi XYbhg`\WJb`k cf_hck UfXg`gdY\JZJYX`ghJbXUfXg"``H\]g`]g`h\Y`ZJbU` UggYgga YbhcZh\Y`\Wti fgY

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K YY_''&-'>i`m!'& 5i[igh	GYa]bUf	: ccX'gYW f]lmž[YbXYfž'UbX'XYjY`cdaYbh H\Y']bhYfgYWjcb'cZh\YgY'hcd]Wg'ZcW gYg'cif UhhYbhjcb'cb'k caYb'g'YWcbca]WUbX'U[f]W'hifU` fc`Yg']b'h\Y'; `cVU``Gcih\ž'UbX'YZCfhg'hc'dfcachY YadckYfaYbhh\fci[\'h\YgY'`]jY`]\ccXg"
K YY_'%\$.') '5i [i gh!'-5i [i gh	GYa]bUf	H\Y'Z ii fY'cZ]bHYfbUh]cbU`XYj Y`cda Ybh +DYei U`]Imžk Y``!VY]b[žUbX h\Y'dcgg]V]`]ImicZ [fYYb'[fck h\ -D h\]g'k YY_'g'gYa]bUfz'h\Y'Z]bU``Zcf'5F HG' +) * žk Y `cc_'ic h\Y'Z ii fY'cZ]bhYfbUh]cbU``XYj Y`cda Ybh''K]h\ U`\][\`mWcbhYghYX'UbX'Yj Yf'Yj c'j]b['dUghž'k \Uh a][\\hh\Y'Z ii fY'cZci f'Z]Y'X'fYgYa V'Y3'H\fci [\ci h h\Y'gYa YghYfz'UbX']bXYYX'h\Y'Ybh]fY'XYj 'gh X]Yg a U'cfz'k Y'\Uj Y'Wcbg]XYfYX'U'fUb[Y'cZX]ZYfYbh UddfcUWXYg'ic XYj Y`cda Ybh''+D'VfcUX'hYfa gz'h\YgY gYY_'UXXfYgg'[`cVU``dcj YfmUbX']bYei U`]Imi]b j Uf]ci g'k Um\$ fz'Wcbg bh]cb```m

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7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

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- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

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- fYei Yghg Zcf Yl hYbg]cb/
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- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhmidc`]M]Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUljcb`]b`l\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
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