

COMD5000

B; Cgž7]j]`GcW]YhmUbX8YjY`cdaYbh

HYfa 'C bY''##'&\$%-

7ci fgY[·]Cj Yfj]Yk

GHUZZ:7cbHUWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
5bhcbmNk]	U"nk]4 i bgk "YXi "Ui	D'YUgY'Ya U]' hc UffUb[Y'U'h]a Y'hc X]gW gg/d'UW/ Wci fgY'WcXY'UbX]ggi Y']b'gi V'YWh \YUX]b["'Hi YgXUm cf'H\i fgXUm %%\\$\$!%&\\$\$	F cca &) ž ; fci bX: `ccfž A cfj Yb 6fck b 6 X[\$(&' *-*(-\$

GWXcc`7cbHJWhi±bZcfa Uh]cb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

7 ci fgY @YUfb]b[Ci hWca Yg

%" 5 bu'mgY'h\Y'gcWJU'ž'YWcbca]WUbX'ch\Yf'WcbhYl hi U'`W\Ub[Yg'h\Uh\Uj Y'`YX'hc'h\Y'Ya Yf[YbW'`UbX [fck h\'cZ`Uf[Y'bi a VYfg'cZWJ]]'gcWJYhmž'bch'Zcf!dfcZjh'UbX'gcWJU``di fdcgY'`cf[Ub]gUh]cbg/&"

; fci d'k cf_']bWi X]b['[fci d'dfYgYbHJh]cb'UbX'[fci d'fYdcfh!'U'_YmY'Ya YbhcZh\]g'Wci fgY'k]```]bWi XY [fci d'k cf_']bWi fgYbHJh]cb'UbX'[fci d'k cf_']bWifb]b['Yb[U[Ya Ybhk]h\'Ub'B; C'k cf_]b[']b'fYU`'h]a Y'hc'Wcbhf]Vi hY'hc']bhYfbUh]cbU`XYj Y`cda Ybh''H\Y'[fci d'k]``'fYgYUfW\'Ub']ggi Y'hc[Yh\Yfz'dfYgYbhhc'h\Y'WUgg'UbX'B; C'z'UbX'gi Va]hU fYdcfhfYZYWhb['h\Y]f']bg][\hg'UbX'[i]XUbW'''; fci d'k cf_'g_]``g'UfY'YggYbhJU`'hc'k cf_]b[']b'XYj Y`cda Ybh gYhJb[g"

DUdYf'Wf]hjei Y'ZUWj]hUhYg'Wf]hjWU``UbU`mg]g'cZU'di V`]g\YX'dUdYf''H\]g'dfcj]XYg']bg][\hg']bhc'h\Y'dYYf!fYj]Yk dfcWfgg'UbX'\ck '&i fbU``UfhjWYg'[Yhdi V`]g\YX'Ug'k Y```Ug'Vi]'X]b['Wca dYhYbWf']b'YI Ua]b]b['UbX Wf]hjei]b['X]ZYfYbhhmdYg'cZ'&i fbU``UfhjWYg'': cW g]b['cb'fYWfbhmdi V`]g\YX'dUdYfg'U`gc'\Y`dg'ghi XYbhg gYY'\ck 'h\Y'Z]Y`X']g'XYj Y`cd]b['UbX'k \Uh]ggi Yg'UfY'VY]b['X]gW ggYX'''Ghi XYbhg'dfYdUfY'U'Wf]hjWU` gi a a Ufmh\UhVf]b[g']b'ch\Yf'`]hYfUhi fY'UbX'dfcj]XYg']bg][\hg']bhc'k \Yh\Yf'ch\Yfg']b'h\Y'WUgg'g\ci `X df]cf]hjgY'h\Y'dUdYf']b'h\Y]f'Z fh\Yf'fYUX]b['UbX'fYgYUfW\'UWf] jhyg''

±DXYdYbXYbh`YUfb]b['Yl YfW]gYg'! 'gca Y'UW]j]h]Yg'k]```VY'i d`cUXYX'hc'h\Y'A ccX`Y'g]hY'UbX'ghi XYbhg'k]```VY YbWci fU[YX'hc'j]Yk žfYgdcbX'UbX'Wf]h]ei Y'h\YgY']b'h\Y]f'ck b'h]a Yžk]h\'U'j]Yk 'hc'g\Uf]b['t\Y]f']bg][\hg cb`]bY"'H\Y'Wci fgY'XfUk g'cb'bi a Yfci g'Z]`a g'UbX'j]XYcg'UbX'ghi XYbhg'U`gc'[U]b']bg][\hg]bhc'\ck 'hc Wf]h]WU``mj]Yk 'h\YgY'a UhYf]U`g"'

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5XX]h]cbU`'XYhU]`g.

H\]g`YggUmg\ci`X`ZcW g`cb`lk c`]ggi Yg. '%£']bg][\hg`Uf]g]b[`Zfca `h\Y`fYU`!`]ZY`fYU`!h]a Y`ZcW g`cb`Ub`B; C gYY_]b[`hc`UXXfYgg`_Ym]bhYfbUh]cbU``XYj Y`cda Ybh]ggi Yg`UbX`&£'dfcj]XY`Ub`]b!XYdh\`Yl Ua]bUh]cb`UbX Wf]h]ei Y`cZ]bbcj Uh]j Y`ghfUhY[]Yg`h\UhWb`VY`fbf`UfY£'i gYX`VmWJj]``gcWJYhmcf[Ub]gUh]cbgž`]bhYfbUh]cbU`B; C g`cf``cWJ``B; C g`UbX`Wca a i b]hm\VUgYX`cf[Ub]gUh]cbgž'hc`dfca chY`hfUbgZcfa Uhjj Y`XYj Y`cda Ybh]b`U gdYWJZJWWcbhYl hUbX`gYhn]b["'5XX]h]cbU``XYhU]`g`hc`VY`X]gW ggYX`]b`WUgg"

5ggYgga Ybh' .; fci d fYdcfh/ dfYgYbhUhcb

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. '5 'fYdcfriVm'[fci dg'cZ(!*'ghi XYbhg'ZcWig]b['cb'U'gdYW]Z]Whcd]WfY`Uh]b['hc'fYU'!h]a Y'WkU``Yb[Yg ZUW]b['U'gdYW]Z]XXB; C"'Bc'a cfY'h\Ub'(\$\$\$'k cfXg'fk f]hhYb'Vmk\c`Y'[fci d±']bWi X]b['U'ik c'dU[Y 9 | YWi'h]j Y'Gi a a Ufm''K f]hhYb'ZYYXVUW_'k]h\]b'%\$'k cf_]b['XUmg"'''H\]g']g'h\Y'Z]bU``UggYgga Ybh'Zcf'h\Y Wti fgY"

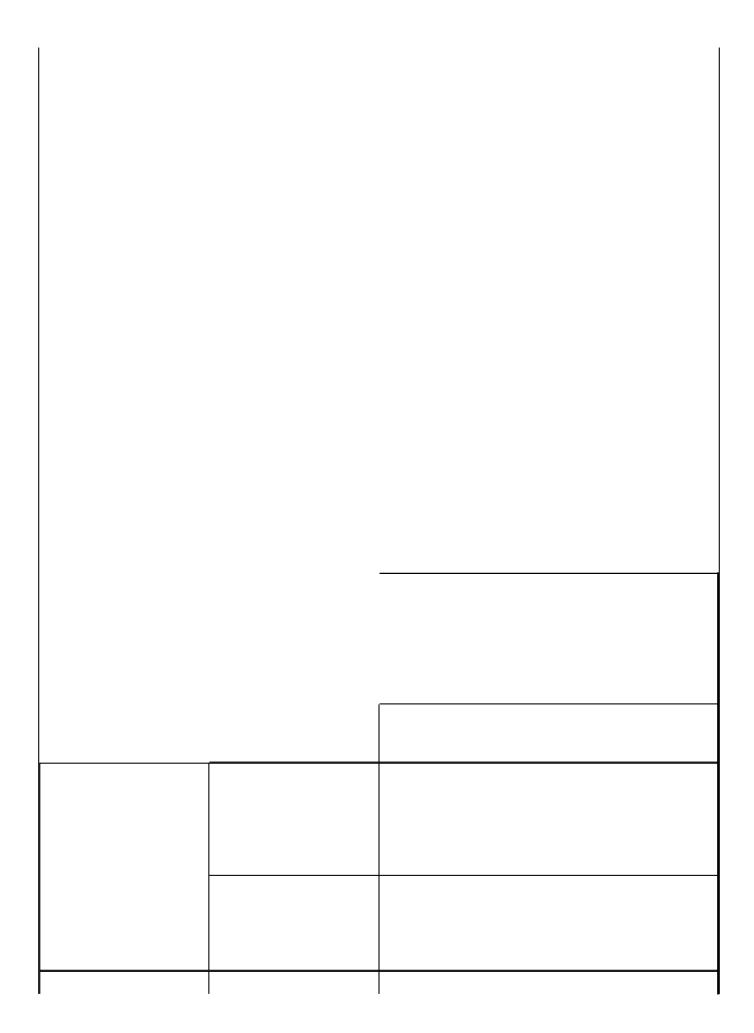
5XX]h]cbU`XYhJ]`g.

; fci dg``]_Y`mhc`VY`cZZci f`gh XYbhg`cb`m'`Hk c`Wca dcbYbhg`!`U`[fci d`fYdcfh'UbX`U`[fci d`dfYgYbhJh]cb`Ug
]Zmci f`[fci d`k YfY`Wcbgi `hUbhg`dfYgYbhJb['hc'h\Y`B; C`cb`k \]W\`k Y`UfY`ZcW g]b['!`h\]g`k]```hU_Y`d`UW\
Xi f]b['ci f`]bhYbg]j Y`cb`Gi bXUm5df]``%("'6ch\`h\Y`fYWca a YbXUhJcbg`UbX`h\Y`a cXY`cZdfYgYbhJhJcb`UbX
fYdcfhlk f]hJb['g\ci `X`UW_bck `YX[Y`UbX`hU_Y`bchY`cZh\Y`WcbhYl h! 'cddcfh b]hJYg`UbX`WcbhfU]bhg`! 'hc
dfca chY`hfUbgZcfa UhJj Y`XYj Y`cda Ybh'K f]hhYb`fYdcfhcb`VY\U`ZcZh\Y`[fci d`a i ghiVY`gi Va]hhYX`Vmh\Y`Xi Y
XUhY`!`U``a Ya VYfg`cZh\Y`[fci d`a i ghi d`cUX`hc`Hi fb]hJb`h\Y`gUa Y`fYdcfh'``

 $5 \ \ dYYf'UggYgga \ YbhcZdUff]W]dUh]cb']b'h\Y'[fci \ d'UWf]j]h]Yg'k]'``U``ck'a \ cXYfUh]cb'cZh\Y'5ggYgga \ Ybh'' 'a \ Uf_h\UhYUW\'dYfgcb'k]'``TYWY]j'Y''$

Gi Va]gg]cb`bchYg.DfYgYbhUh]cb`k]```VY`Xi f]b[`]bhYbg]j Y`cb`%(`5df]`"`; fci d`fYdcfhWUb`hc`VY`dfYgYbhYX VmXUhY`UbX`ha Y`qh]di `UhYX"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYH]b[. Hi]g Ugg][ba Ybh]g gi Va]hYX hi fci [in Hi fb]h]b UbX ghi XYbhg WUb gYY Hi fb]h]b g]a] Uf]hm fYdcfhg"$



		7 UgY 'gh XmX]gW gg]cb 'VUgYX 'Ufci bX 'df]cf 'Z] 'a gWfYYb]b['UbX 'WUfYZ `'bchY!hU_]b[" 8 YhU] 'g'cb 'A ccX`Y	
	±bfYbg]jY	#blYbg]j Y'WUgg'k]``VY'\Y`X'cb'Gi bXUm'%('5df]``! Zfca '-''' \$Ua 'hc'(''' \$da ']b'A cfj Yb'6fck b'; *" ; fci dg'k]```dfYgYbhh\Y]f'[fci d'k cf_'fYgYUfW\'UbX UXj]\W'cb'h\]g'c\W\Ug]cb'fH67\t''5'g\UfYX'`i b\W'k]`` U'gc'VY'\Y'X'! YUW\'dYfgcb'hc'Vf]b['U'g\UfY'd`U\W'' : i fh\Yf'XYhU]`g']b'WUgg''	
K_YY_''%) '5df]`'!'&% 5df]`	GYa]bUf	; ccX'dfUWfJWY'YI Ua d`Yg`]b`B; Cg`UbX 8YjY`cda Ybh'	
	5ggYgga Ybh	Gi Va]h[fci d'fYdcfhUbX'Yb[U[Y']b'Wcbgi 'HJHj Y X]gW gg]cb'k]h\'5A: ']b'WUgg"'; fci dg'a UmdfYgYbh cf'Z]bX'U'a cfY']bbcj UHj Y'a YUbg'cZg\Uf]b['H\Y]f]bg][\hg'k]h\'h\Y': ci bXUH]cb"'7f]hYf]U'UbX'[i]XUbW cb'A ccXY'UbX']b'WUgg"	
K YY_'‱'&-'5df]`'!'% A Um	5ggYgga Ybh	Gi Va]h'ZbU``fYZYWfj Y`YggUm! Xi Y`cb`&- `5df]``Vm) da	

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DfYgWf]VYX F Ygci fWyg

GYY'AccX`Y'UbX'@Y[Ubhc'`]b_g

FYWca a YbXYX FYgci fWYg

GYY'AccX`Y'UbX'@Y[Ubhc'`]b_g

Cb`]bY'6`c[g#DcXWUghg.

A cgh'V`c[g'UfY']bZcfa U``cd]b]cb'YX]rcf]U`gz'k f]hYb'ei]\w_`mUbX`cZhYb'k]h\ci hgi VghUbh]U``gi ddcfh]b[Yj]XYbWY"'H\YmWJb'VY'Ub'YI WY``Ybhgci fWY'cZj Yfmh]a Y`mUbU`mg]g'UbX`WJb'dc]bhmci ']b'h\Y'X]fYW]cb'cZ h\Y``UhYgh]bZcfa Uh]cb'cb'dc']\wn]ggi Yg''' \night]g']a dcfhUbhhc'bYj Yf'fYUX'U'V`c['i bWf]h]\w]\mm'' \night]b\wi i fU[Y'mci 'fc hfmhc'Zc``ck 'Uh`YUghcbY'cb'U'ZfYei YbhVUg]g'hc'[]j Y'mci 'gca Y'fYU'!`]ZY'i bXYfghUbX]b['cZh\Y`'UhYgh XYVUhYg']b'XYj Y`cda Ybh'' \night]\wy Y'U`gc']b\wi XYX'cbY'dcX\wJghUbX'cbY'k YVg]hY'k]h\g\cfhihU`_g'Zcfh\cgY cZmci 'k\c'k Ubhgca Yh\]b['hc'']ghYb'hc'QfYUhk\Yb'cb'di V`]\whfUbgdcfhQcfk Uh\X']bghYUX'cZ^fghfYUX]b[.

• 8Yj Y`cda YbhDc`]\mn7YblfY. \https://whty.jdc\]\mn7YblfY. \https://whi.wfuk.zcfx"Ubi "YXi "Ui #

- 9i fcdYUb '>ci fbU`'cZ8Yj Y`cda YbhF YgYUfW
- ; YbXYf'UbX'8Yj Y`cda Ybh
- ; `cVU``; cj YfbUbW
- #bh/fbUhjcbU`Cf[Ub]nUhjcb
- >ci fbU`cZ8Yj Y`cda YbhGh X]Yg
- >ci fbU`cZ<i a Ub'8Yj Y`cda Ybh
- >ci fbU`cZ±bhYfbUhJcbU`8Yj Y`cda Ybh
- >ci fbU`cZDYUgUbhGh X]Yg
- >ci fbU`cZDYUWVi]`X]b['UbX'8Yj Y`cda Ybh
- CIZcfX'8YjY`cdaYbhGhX]Yg
- Dfc[fYgg]b'8YjY`cda YbhGh X]Yg
- H\]fX'K cfX'EiUfhYfm
- K cfX'8YjY`cda Ybh

Mci 'k]```U`gc`VYbYZhZca``VYWca]b[`ZJa]`]Uf`k]h\; cc[`YGW\c`Uf`f\hd.#gW\c`Uf"[cc[`Y"Wca Ł'Ug'U'_YmgYUfW\'Yb[]bY`Zcf`UWUXYa]Wdi V`]WUh]cbg'UbX`fYdcfhg"'Mci 'WUb`gYhi d`h\Y`dfYZYfYbWYg'hc``]b_'hc'h\Y I BGK '@|VfUfmYj Yb'k\Yb'mci 'UfY'bchcb'WJa di g"

; c'hc'; cc['Y'GW\c'Uf2'gYht]b[g2'`]VfUfm']b_gž'UbX'YbhYf''l b]j Yfg]hmcZBYk 'Gci h\ 'K U'Yg'']b'h\Y'Vcl 'Zcf "@\VfUfm'.' Mci 'Wub'g][b'i d'Zcf'HUV'Y'cZ7cbhYbhg'fHC7Ł'5`Yfhg'Zfca 'h\Y'\ca YdU[Yg'cZfY`Yj Ubh'ci fbU`gž hc'fYWY]j Y'Ub'Ya U]''k\YbYj Yf'bYk 'Ufh]WYg'UfY'di V`]g\YX']b'h\Uh'ci fbU'''>ci fbU'`k YVg]hYg'k]'``cZhYb 'WUffm]bZcfa Uh]cb'cb'h\Y'a cghj]Yk YX'UbX'a cghW]hYX'Ufh]WYg/h\YgY'UfY'`]_Y`mhc'VY']bhYfYgh]b['UbX'cZhYb]bZi Ybh]U``Wcblf]Vi h]cbg''; cc['Y'GW\c'Ufk]''Ugc'dc]bhmci 'hc'Ufh]WYg'h\Uh\Uj Y'W]hYX'U'dUfh]W'\Uf1JWY UbX'\YbW'k]```VY'fY`UhYX'hc'h\Y'hcd]W'

7ci fgY 9j U'i Uh]cb UbX 8Yj Y'cda Ybh

Gh XYbhZYXVUW_`\Ug`Wcblf]Vi hYX`UWhj Y`mhc`fYg\Ud]b[`h\Y`Wci fgY`hc`a U_Y`]ha cfY`Yb[U[]b[`UbX`hc`cZYffYU`!ha Y`]bq][\hg`]bhc`U`XYj Y`cda Ybhcf]YbhYX`B; C`VUqYX`]b`5i qlfU`]U`Vi hk cf_]b[`]bhYfbUh]cbU``m"`

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fWYg'k\]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[`h\Y`″gYWcbXUfm″gci fWY`Zfca 'k\]WX`_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fuqlb[žqi a a Uf|q]b[žYggUmk f]h]b[UbX h]a Y a UbU[Ya Ybh
- 5 dd fodd f]Uhr dyf c lloub llfor i lloub llf

5 WUXYa]W±bZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb'fY`UhJb['hc.