

ARTS4268

HYfa 'C bY''##'&\$%

7ci fgY[.]Cj Yfj]Yk

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Giaa UfmcZh\Y 7ci fgY

 $BchY. H\ [g']g'U'\% I C7'WcifgY''Mcik]``Ybfc``]b'* I C7']b'YUWX'cZhk c'giWW'gg]j Y'hYfa g'k]h\ 'U'fYgi`h fYdcfhYX'Vm'U'g]b[`Y'[fUXY'Uh'h\Y'YbX'cZh\Y'gYWcbX'* I C7'WcifgY''$

H\]g']g'U'WcfY'Wci fgY'Zcf'<cbci fg'WUbX]XUHYg']b'h\Y'GW\cc``cZGcW]U``GW]YbWrg'UbX'h\Y'a U]b'ZcW'g'k]```VY h\Y'ei Ygh]cb'cZa Yh\cXc`c[m''9UW\gYa]bUf'k]````cc_'UhX]ZZYfYbh'UddfcUW\Yg'hc'fYgYUfW\'UWfcgg'h\Y'gi V!X]gW]d']bYg'UbX'k]```YI d'cfY'k\Uhk Y'a][\h\W\``h\Y''gc'k\Uhf'cZU'dUfh]W``Uf'a Yh\cXc`c[mg'UbU`mf]WU``Yj YfU[Y''K\mXc'h\Y'GcW]U``GW]YbWrg'VfYU_']bhc'a mf]UXž'Wca dYh]b['a Yh\cXc`c[]Yg3'7 Ub'k Y'XYgWf]VY ci f'a Yh\cXc`c[]Yg'Ug'gW]Ybh]Z[Wz'UbX']Zbchz'h\Yb'\ck Xc'k Y'VYghgYW'fY'U'bch]cb'cZUWW'fUWmUbX i gYZ\`bYgg3'<ck 'XcYg'ci f'i bXYfghUbX]b['cZYj]XYbW''UWh'U`mW\Ub[Y'ci f'fYgYUfW\'a Yh\cXg'Ug'k Y``'Ug h\Y'ci hWca Yg'UbX'WcbWi g]cbg'h\Uhk Y'a][\hfYUW\3

7 ci fgY @YUfb]b[Ci hWca Yg

 $\%'' 5 dd mWf hWU`h b_b [UbX'UbU`mhWq_]`q$

&" 9ZYWIJ Y`mXYd`cmU'dUffJW``Uf'I\Ycfmcf'a YI\cXc`c[]WU``UddfcUW.

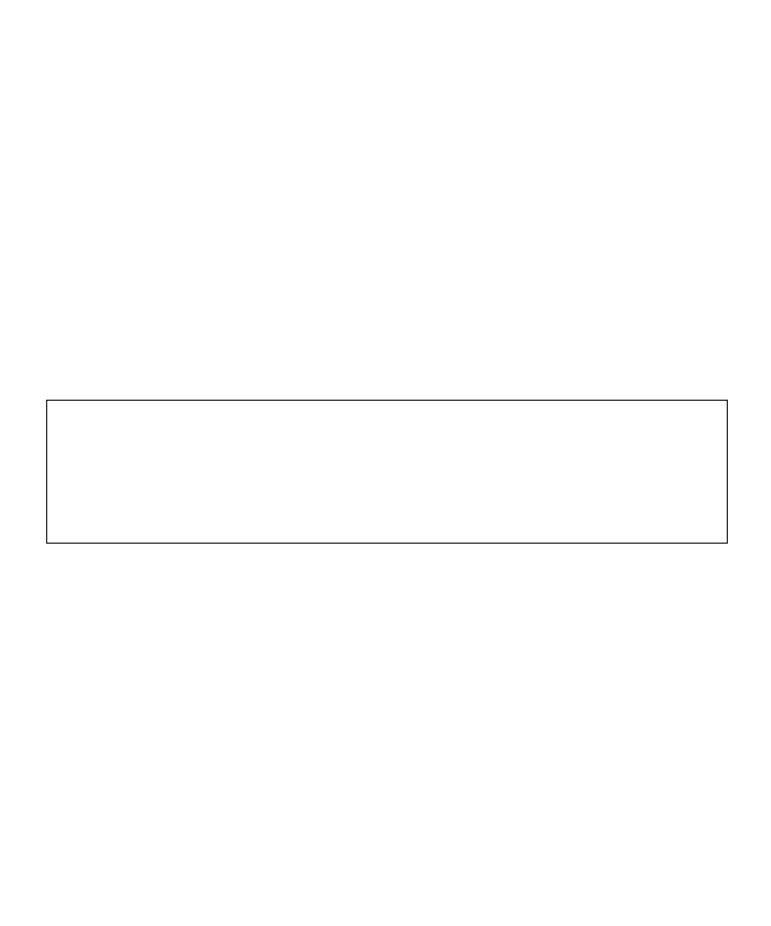
' "8 Yg][b'UbX'gi gHJ]b'Ub'Uf[i a Ybhzcf'dc]bhcZj]Yk

("→YYbr|ZmUbX'Udd'mrh\Y'Yh\]WU``fYei]fYa Ybrg'UbX`fYgdcbg]V]`]r]Yg'cZfYgYUfW\Yfg'`

HYUWX]b[GhfUhY[]Yg

GYa]bUfg`k]``VY`WcbXi WfX`Vmh\Y`WccfX]bUhcf`k]h\`]bdi hZfca `ch\Yf`gdYW]U`]ghhYUW\]b[`ghUZZhc`[i]XYghg`h\fci [\h\Y`dfcWfgg`cZWcbXi Wf]b[`fYgYUfW\`Uh\cbci fg``Yj Y`žk cf_]b[`k]h\`U`gi dYfj]gcf`UbXd`Ubb]b[`UbX`YI YW h]b[`Ub`]bXYdYbXYbhfYgYUfW\`dfc`YWf

@]VfUfmfYgcifWgZcf`\cbcifg`ghiXYbhg`k]``VY`YId`U]bYX`Vmh\Y`cihfYUW\``]VfUf]Ub`hc`Ugg]ghighiXYbhg`k]h\fYgYUfW\]b[`h\Y]f`h\Yg]g`hcd]W



V]V`]c[fUd\mVi h']bWi g]j Y`cZYj Yfmh\]b['Y`gY"'Mci `k]````cgY`) ı 'dYf`%\$\$`k cfXg`cj Yf`h\Y`%\$\$`k cfX`]a]h' H\Y`ghUbXUfXg`U[U]bghk \]W\`h\]g`UggYgga Ybhk]```VY`Yj U`i UhYX`UfY`]b`h\Y`fi Vf]W*Uj U]`UV`Y`cb`A ccX`Y"

Mci 'a i ghigi Va]himci fiX]gW gg]cbidUdYfihciHi fb +hbi]biA ccX`Y`Vm' da i: f]XUm'%) 'A UfWX'' = k]``bch UVWYdhigi Va]qq]cbq'VmYa U]''''

$$\label{eq:hambound} \begin{split} &\text{H} \setminus Y : \text{UW `hm`Uh' 'dYbU`hm']g'} \right) \\ &\text{I 'cZh} \setminus Y \text{'hchU`'dcgg]V'Y 'a Uf_g' Zcf'h\ Y 'hUg_' Zcf'YUW\ 'XUmcf'dUfhiXUmih\ Uhih\ Y \\ &\text{k cf_']g'`UhY'' @UhYbYgg']bWi XYg'k YY_YbXg'UbX'di V`]W\ c`]XUmg'' \end{split}$$

H\Y'X]gW gg]cb'dUdYf'k]``VY'a Uf_YX'#%\$\$'UbX'k]``Wcbghhi hY'&\$i 'cZmci f'[fUXY'Zcf'h\]g'Wci fgY''

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

5ggYgga Ybh& 8]gW gg]cb DUdYf

GhUfh'XUhY.

@Yb[h\. *\\$\$\$ k cfXg

8YHJ]`g. `Ghi XYbhg`gi Va]h`cbY`&\$\$\$`k cfX`]bX]j]Xi U``X]gW gg]cb`dUdYf`cb`U`g]b[`Y`gdYVJZ]YX`Wci fgY`h\Ya Y Zcf`UggYgga Ybh'`Ghi XYbhg`fYWY]j Y`k f]hhYb`ZYYXVUW_`UbX`U`bi a Yf]WU``[fUXY`k]h\]b`hk c`k YY_g`cZ gi Va]gg]cb"

5XX]h]cbU`XYhU]`g.

H\Y`gYWcbX`X]gW gg]cb`dUdYf`]g`U`%\$\$\$`k cfX`YggUmcb`U`WcbW\dhfY`Yj Ubhhc`mci f`h\Yg]g"`G]a]`Uf`hc`h\Y YggUmg`fYUX`]b`hcd]W' ž`mci `k]```XYj Y`cd`Ub`Uf[i a Ybh]b`fY`Uh]cb`hc`k \mih\Y`h\fa `WUb`VY`Wcbg]XYfYX`U WcbW\dhžYI d`cfY`h\Y`Wcbh\gh\X`bUhi fY`cZh\Uh`WcbW\dhžUbX`YI d`U]b`]hg`dfcXi Whj YbYgg`]b`mci f`UfYU`cZqh`Xm''

5XX]h]cbU`'XYhU]`g.

H\Y'h\]fX'UbX'Z\bU``UggYgga Ybh'HJg_'Zcf'h\]g'Wci fgY']g'h\Y'fYgYUfW\'dUdYf"'\\$'h\]g'dUdYfž'mci 'a i gh UXXfYgg'h\Y'Zc``ck]b['h\fYY'ei Ygh]cbgž'k\]W\'UfY'h\Y'gUa Y'h\fYY'hcd]Wg'h\Uh'h\Y'[i Ygh'dfYgYbhYfg'\Uj Y UXXfYggYX'YUW\'k YY_'h\fci [\ci h'h\Y'gYa YghYf.

- %" K \UhiX]gWJd`]bUfmUbX`gW\c`Uf`miffUX]fJcbg`#dYfgdYWJj Yg`#gYbg]V]`]hJYg`Xc`mci `Vf]b['hc'h\Y fYqYUfW\'h\Uhimci 'k\]```Xc'h\]q`mYUf3`
- ' " < ck 'Xc'h\YgY``YUX'hc'dUfh]W``Uf'k Umg'cZi bXYfgHJbX]b['UbX'fYgYUfW\]b['h\Y'gcW]U``k cfX3'K \UhUfY'h\Y'ghfYb[h\g'UbX``]a]hUh]cbg'cZmci f'X]gW]d`]bUfm#gW\c`Uf`mdYfgdYWf]j Y3'

Mci 'g\ci `X'YI d`cfY'h\YgY'ei Ygh]cbg'UbX'fYZYWfi dcb'h\Ya 'VYZcfY'k f]h]b['i d'U'h\ci [\hZ ``UbX'dYfgcbU` mYhgW\c`UfmdUdYf"9UW\ 'gYW]cb'g\ci `X'XYj Y`cd'Ub'Uf[i a Ybhcf'gYhcZUf[i a Ybhg'UbX'h\Y'dUdYf'Ug'U k\c`Y'g\ci `X'VY'Vch\ 'Wc\YfYbh'UbX'Wc\Yg]j Y"'K Y'k]```gdYbX'h]a Y']b'WUgg'X]gW gg]b['\ck 'hc'UddfcUW\ h\]g'UggYgga Ybhz'UbX'Z fh\Yf'fYgci fWfg'UfY'Uj U]`UV'Y'cb'A ccX'Y"'

 $\label{eq:higher} \begin{tabular}{ll} High \$

5 https://bxubwww.Fyei]fya Ybhg

 $\label{eq:continuity} D`YUgY`bchY`h\Uh`YWh fY`fYWcfX]b[g`UfY`bchUJ U]`UV`Y`Zcf`h\]g`Wch fgY"`Gh XYbhg`UfY`ghfcb[`mYbWch fU[YX`hc UhhYbX`U```WUggYg`UbX`WcbhUWh\Y`7 ci fgY`5i h\cf]mmhc`a U_Y`U`hYfbUh]j Y`UffUb[Ya Ybhg`Zcf`WUggYg a]ggYX"$

7 ci fgY GWX YXi `Y

J]Yk Wugg'ha Yhuv'y

H]a YHJV`Y

8 UhY	Hmd Y	7 cbhYbh
Hcd]W% '9d]ghYa c`c[m UbX`A Yh\cXc`c[ml&* ': YV &\$%-		
Hcd]W&:"K cf_]b["k]h\"h\Y fgY`Zfi'dcg]h]cbU`]rm!%& A UfW\"&\$%		
Hcd]W" K cf_]b[k]h\ WcbWYdrg!&* A UfW. &\$%		
Hcd]W(."K cf_]b["k]h\ h\Ycf]Yg!-"5df]"&\$%		
Hcd]W)		
Hcd]W*."Gh Xm]b["HYI hg!" >i bY"&\$%		
Hcd]W+. 'Gh Xm]b[±bX]j]Xi U'g!%+ '>i bY &\$%		
Hcd]W, .'Gh Xm]b[gcV]Yhn!%>i `m&\$%		
Hcd]W:GhXm]b[a YX]U!%):>i `m&\$%		
Hcd]W%\$."H\Y f]fLfY`Yj UbWY"cZ X]gWJd`]bYg!'\$">i `m&\$%		

FYgci fWyg

DfYgWf]VYX F Ygci fWfg

8 YHU]`g`cZU```H\Y`fYUX]b[g`UfY`cb`AccX`Y"`GHi XYbhg`UfY`YI dYWYX`hc`Wcad`YhY`U``fYUX]b[g`YUW\`kYY_"D`YUgY`bchY`H\UhifYUX]b[g`aUmW\Ub[Y`id'hc`lkc'kYY_g`df]cf`hc`WUggž`gc`d`YUgY`W\YW_`]b`hc`AccX`Y`df]cfhc`Wcad`Yh]b[mcif`fYUX]b["

FYWca a YbXYX FYgci fW/g

FZWdW[e`a UadWfW] f XadfZ[e UagdeW SfZagYZ efgVW fe_Sk X[V fZWXa^ai [Y geWg.]

8 "'MJbck 'UbX'D"'GWkk Uffm!G\YUž'YXg'f&\$%(E';`fWbbdWf\$f[a`S`V? WfZaV, 7_b[d[U\$^D WeW\$dJZ? WfZaVeS`V fZW;`fWbdWf\$f[hWFgd` \dot{z} '& \dot{z} 'XXb \dot{z} '@cbXcb.'A "'9"'G\UfdY"

5XX]h]cbU`fYUX]b[fYgci fWg'UfY`cWUhYX`]b h\Y`UggYgga YbhgYWfjcb'cZA ccX`Y

7 ci fqY 9j U'i Uhcb UbX 8 Yj Y'cda Ybh

G][b]Z]WUbhYYa Ybhg`cZh\]g`Wci fgYž`]bWl X]b[`UggYgga YbhUbX`k YY_`mhcd]Wgžk YfY`WkUb[YX`Zc``ck]b[gh XYbhZYYXVUW_`]b`&\$% "`=k]``gYY_`]bZcfa U``ZYYXVUW_`Zfca 'mci 'hkfci [\ci hh\Y`Wci fgY`gc'h\UhkY`Wb Ybgi fY'h\Uhih\Y`h]a Y`gdYbhhc[Yh\Yf`]g`a cfY`dfcXi Whj Y`Zcf'mci "`=k]```U`gc`Ug_`mci 'hc'Wca d`YhY'h\Y a m9I dYf]YbWl`gi fj YmUhh\Y`YbX`cZhf]a YghYf`&ž`gc'h\Uhh\Y`Wci fgY`WUb`VY`Z fh\Yf`]a dfcj YX`Zcf`Z hi fY \\Cfng"`=YbWci fU[Y`mci 'hc'hU_Y`fYgdcbg]V]`]mmZcf'h\Y`fi bb]b[`cZh\]g`Wci fgY"`=Ua 'cdYb'hc`\YUf]b[`mci f qi [[Yqh\cbg`Ug`hc'h\]b[q`mci 'k ci `X``]_Y'hc'Xc'Xi f]b[`ci f'h]a Y'hc[Yh\Yf''

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWUb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU``mWcdm]b['Zfca `U`gci fW'k]h\ci hUW_bck `YX[Ya Ybh'

 $I BGK `[fci dg'd`U[]Uf]ga `]bhc'h\Y'Zc``ck]b[`WUhY[cf]Yg.$

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/