

ARTS2248

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± WfY UgYgʻ]bʻX]gUghY fgʻUʻcb[ʻk]h\ʻh\ʻY W ffY bhiUbXʻZ hi fYʻdfYX]WfXʻ]a dUWigʻcZW]a UhYʻW\Ub[Yʻ\Ugʻa YUbh h\UhX]gUghY fa UbU[Ya YbhiUbXʻX]gUghY fʻf]g_ʻfYXi Wfjcbʻ\Uj YʻYa Yf[YXʻUgʻUʻ_YmdfcʻYWhiUbXʻdc`]WmUfYUgʻʻH\]gʻWci fgYʻdfcj]XYgʻmci ʻk]h\ʻWfjh]WUʻdYfgdYWfj YgʻhcʻYI Ua]bYʻh\]gʻbUhi fUʻX]gUghY fgʻ#gcWJY mi]bhYfZJWYʻʻH\YYa d\Ug]gʻcZh\YʻWci fgYʻ]gʻZcWʻgYXʻcbʻbUhi fUʻX]gUghYfgʻVY]b[ʻVYhYfʻi bXYfghccXʻUgʻgcWJUʻX]gUghYfg k]h\ʻbUhi fUʻHJ[[YfgʻʻH\i gž]bʻh\Y`Wci fgYʻmci ʻk]``XYj YʻcdʻUbʻi bXYfghUbX]b[ʻcZ\ck ʻZcfa gʻcZgcWJUʻji ʻbYfUV]]mhcʻ\UnUfXgʻYa Yf[YʻUbXʻg\UdYʻgi VgYei YbhX]gUghYfj]a dUWigʻʻMci ʻk]``UʻgcʻYI Ua]bYʻ\ck bUhi fUʻX]gUghYfgʻUfYʻX]gfi dhji YʻYj Ybhgʻh\UhWUbʻWf]hJWUʻmʻc'hUbXʻg\UdYʻZi hi fYʻgcWJUʻžYWcbca]WUbX dc`]hJWUʻci h\cka Ygʻ]bʻh\YʻWcbhYI hcZ[cj YfbUbWf`UbX'dc`]hJWgʻʻ±bʻh\YʻZ[fgh\UʻZcZh\Y`Wci fgYʻmci 'k]``YI Ua]bYʻX]ZYfYbhih\Ycf]YgʻUbXʻZfUa Yk cf_gʻZcfʻi bXYfghUbX]b[ʻX]gUghYfgʻʻH\]gʻk]``dfcj]XYʻUʻWf]hJWUʻdʻUbXcfa ʻZcfʻh\YʻgʻWcbX'dUfhcZh\Y`Wci fgYžk\YfYʻmci 'k]``ZcW gʻcbʻX]ZZYfYbhih\Ya UhJWUfYUgʻcZX]gUghYfg [cj YfbUbWf`UbX`gcWJYhmži g]b[ʻWUgYʻghi XmX]gUghYfgʻZfca ʻUfci bXʻh\Y`k cf`X''

7 ci fgY @YUfb]b[Ci hWca Yg

%" 9 I d`U]b`\ck 'bUhi fU``X]gUghYfgž'gcWjYhmž'XYj Y`cda Ybhidfc[fUa gž'UbX'dc`]WhiUfY`]bhYffY`UhYX" &" 5ddfU]gY'_YmUi h\cfg'UbX``]hYfUhi fY'X]gWi gg]b['X]gUghYfg'Zfca 'U'Wf]hJWU`gcWjU`gWjYbWY'UbX\i a Ub]hYg'dYfqdYWhi Y"

- ' " 5dd`mYb\UbWX``YUfb]b[ž'UbU`mt]Wž'UbX`Wca a i b]WUt]cb'g_]``gž'tc']btYfdfYh`]b_g'VYtk YYb'gcW]Ytm UbX`X]qUqtYfq
- (" '5 bU`mgY 'X]gUghY f`WUgY 'ghi X]Yg`Zfca 'Ufci bX'h\Y'k cfX'k]h\'U`fUb[Y`cZX]ZZYfYbhh\YcfYh]WU` ghUbXdc]bhg"

HYUW]b['GhfUhY[]Yg'

H\Y`gi V^YWik]```i gY`U`V`YbXYX``YUfb]b[`'UddfcUW\/k]h\']bglfi Wi]cb`cWW ff]b[`']b`!\\Y`WUggfcca 'f|YWi fY`UbX h\ hcf]U`gŁž`U]XYX`k]h\`cb`]bY`Yb[U[Ya Ybhi\\fci [\'Ugg][ba Ybhgž`X]gW gg]cbg`UbX`UWij]hjYg"`H\Y`cb`]bY Ybj]fcba Ybhfh\fci [\`a ccX`YŁ`]g`U`bUh fU``YI hYbg]cb`cZlfUX]hjcbU``WUggfcca '`YUfb]b["'5```UggYgga Ybhg

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Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

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8 YHJ] g.

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5 Zbu``YggUmf&z\$\$\$'k cfXgŁ']b'k \]W\`gh XYbhg`Ubgk Yf'cbY'cZU'gY`YWhbi a VYf'cZei Ygh]cbg"

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7 ci fgYgʻUfYʻdYf]cX]\\\U``mfYj]\\K YX\\`UbX`ghi XYbhg'`\ZYYX\\U_`]gʻi gYX`hc`]a dfcj Y`h\Ya\\``: YYX\\U_`]g [\Uh\\YfYX`\\I`ca 'ghi XYbhg'i g]b['a m9\| dYf]\Yb\\\'\'`\\\#i]g`Yb\\ci fU[YX`h\Uhighi XYbhg`\\can d`YhY`h\Y]f`gi fj Ymg`\\m\\U\\\\gg]b['h\Y`d\YfgcbU`]g\YX`k\Y\``]b_j]\U`h\Y`A ccX`Y`\\ci fgY`g]h\Y\"

 $: YYXVUW\underline{z}'k \ Yfh Yf']b'dYfgcb'cf'j]U'Ya \ U] \hat{g}\check{z}'Xi \ f]b['hhY'Wci \ fgY\check{z}']g'YbWci \ fU[YX'']$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
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a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

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: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/