

ARTS3462

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7cbj Ybcfg

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D\cbY.'Ž*%'&'-',)'%*,%

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9a U]`. <u>\U`4 i bgk "YXi "Ui</u>

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7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y'7ci fgY

HYWcifgY]gaU]bmHUi[h]b7]bYgY"

7ci fgY @YUfb]b['Ci hWca Yg

- %" 7 ca dufY`X]ZZYfYbh`]hYfufmubX`uffjgh]WrfuX]hjcbg`h\Uh`\Uj`Y`XYj`Y`cdYX`]bXYdYbXYbhm]b`j`Uf]ci`g`c\WJ`Yg`UWfcgg`h\Y`7\]bYgY!gdYU_]b[`k cfX"`
- &" 7 cbglfi Wh'U`Wf]h]WU`ZfUa Yk cf_'hc'h\ci [\h] ``mX]gW gg'h\Y`]ggi Y`cZ7\]bUfgŁ'UbX`7\]bYgYbYgg"
 " =XYbh]Zmh\Y`X]ZZYfYbWrg`]b`[YbfYg`fZ]Wh]cbžXfUa Už'W]bYa UŁ'UbX'bi UbWrX'j Uf]Uh]cbg'f1]b[i]gh]Wz
 qcW[c!\]qhcf]WU`ž'dc`]h]WU`Ł']b`Ufh]qh]WdfcXi Wh]cbg"
- ("FYgdcbXfYgdYWZ")mfk f]hhyb"ubXj YfVu'E]b"dfcXi Wfj Y"uWuXYa]WX]gW gg]cb"

HYUWX]b['GhfUhY[]Yq'

7`Ugg`XY`]j Yfmk]``]bWi XY`ZJ\WY!nc!ZJ\WY`UbX`cb`]bY`U\W]j]hJYg"'\Mci `k]``Yb[U[Y`]b`\WcgY`fYUX]b[`UbX`j]Yk]b[cZgY`Y\Wik cf_g`cZZJ\W]cbžXfUa U`UbX`\W]bYa U`nc`YI Ua]bY`]ggi Yg`cZfYdfYgYbhUh]cb`UbX`fYdfYgYbhU\]`]mž]XYbhJhmUbX`\W`hi fU``dc`]hJ\\gž`cb[]b[`UbX`VY`cb[]b[ž'U\Wcgg'h\Y`7\]bYgY!gdYU_]b[`k cf`X"`H\YgY`_YmhYI hg k]```VY`gY`Y\WhYX`k]h\`Ub`YmY`hck UfX`h\Y`\Wca d`YI `fY`Uh]cbg\]dg`VYhk YYb`h\YgY`X]Ugdcf]\Wg]hYg`UbX a U]b`UbX`7\]bU"`C b`ncd`cZh\YgY`df]a Ufmgci f\WgžX]g\W gg]cb`]b`YU\W\`k YY_`k]```VY`gi dd`Ya YbhYX`Vm gY\\cbXUfmihYI hg`h\UhX]g\W gg`\W`hi fU``h\Ycfmždc`]hJ\\g`cZ`\Ub[i U[Y`UbX`]XYbhJhmih\Uh`\Uj Y`VYYb`[U]b]b[]b\WY\Ug]b[`]a dcfhUb\W`]b`U\UJXYa]\WX]g\\ci fgY`cZ`\Uh\"`

5ggYgga Ybh

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A ccX`Y`8]gW gg]cb`	(\$1		

9 UWX 'gh XYbh']g 'fYei]fYX 'hc 'gi Va]h('A ccX`Y 'Dcghg 'UbX 'UWhUg 'U '8]gW gg]cb '@YUXYf 'Uh`YUghcbW/" '' Mci a Umbchgi Va]h'U 'A ccX`Y 'Dcghcb 'h\Y 'k YY_mci 'UfY 'UWh]b['Ug '8]gW gg]cb '@YUXYf"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& 9ggUmDfYdUfUh]cb

8YHJ]`g.

Dfcj]XY`U`) \$\$!k cfX`YggUmci h`]bY`UbX`V]V`]c[fUd\mk]h\`bc``Ygg`h\Ub`%\$`]hYa gž`]bWi X]b[`Uh`YUgh' `Vcc_g cb`mci f`YggUmhcd]\V\3+bX]j]Xi U`k f]hHYb`ZYYXVUW_idfcj]XYX`VYZcfY`Z]bU``YggUm'

Hi fb]h]b gYh]b[. 'H\]g ']g 'bch'U 'Hi fb]h]b 'Ugg][ba Ybh

5ggYgga Ybh' . '9ggUm

8YHJ]`g.

K f]hY`U`fYgYUfWX`YggUmcb`U`WXcgYb`hcd]WcZUddfcl]a UhY`m&z\$\$\$`k cfXg"3`\bX]j]Xi U``ZYYXVUW_`dfcj]XYX"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$

5 hyb XUb WY F Yei]fYa Ybhg

5 HY b X Ub W' c ZgYa]b Ufg']g'a Ub X Uhc fm']b 'h]g' Wci fgY''' 5 HY b X Ub W' k]```VY 'hU_Yb 'Uh'h Y 'VY[]bb]b['c ZY UW WUgg"'' Hi fb]b['i d'hc 'WUgg'a cfY 'h Ub') 'a]bi HYg'`UhY']g 'Wcbg]XY fY X 'Ub'i bY I W gY X 'UVgY b W''' I bY I W gY X UVgY b W''' Za 'a cfY 'h Ub''% i cZgYa]b Ufg'k]``fYgi 'h]b 'h Y 'Uk Uf X 'U ZU]`'[fUXY''' 6 Y WUi gY 'h]g']g U'gYa]b Ufž'mci ''UfY 'YI dY WY X 'hc 'UWN j Y 'm Yb [U[Y'k]h 'WcfY 'Wci fgY 'WcbhY b hž'd Ufh j W Ugg X]g W gg]c b 'Vm']ghYb]b['Uh'Y bh j Y m Ub X 'fYgdcb X]b['fYgdY W \text{ `mhc'mci f'WUgga Uh'Yg''' H \ YgY'k]``Yb UV'Y mci 'hc' Uh'U]b.

- %" 7 ca dUfY`X]ZZYfYbh`]hYfUfmUbX`Uff\gr\WhfUX]\\deltacbg`h\Uh\\Uj\Y`XYj\Y`cdYX`]bXYdYbXYbhm`]b`j\Uf]ci\g\`c\\U\Yg`U\\fogg`h\\Y`7\\]bYgY!\gdYU_]b[\ k\cfX''\
- &" 7 cbgffi Wh'U'Wf]h]WU``ZfUa Yk cf_'hc'h\ci [\hZ\``mX]gW gg'h\Y']ggi Y'cZ7\]bUfgŁ'UbX'7\]bYgYbYgg"
- '"⇒XYbhjZmh\Y`X]ZZYfYbWYg`]b`[YbfYg`fZ]Wh]cbž`XfUa Už`W]bYa UŁ`UbX`bi UbWYX`j Uf]Uh]cbg`f1]b[i]gh]Wz gcW]c!\]ghcf]WU`ž'dc`]h]WU`Ł`]b`Ufn]gh]WdfcXi Wh]cbg"
- ("FYgdcbXfYgdYWMZ" `mfk f]hh/b UbX' j YfVU'E]b dfcXi Whj Y UWUXYa]WX]gW gg]cb"

7 ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YHJV'Y

H]a YtUV`Y

8 UHY	Hmd Y	7 cbhYbh	
K YY_'%'%)': YVfi Ufm!'%- : YVfi Ufm	GYa]bUf	7 ci fgY`±blfcXi W¶cb	
K YY_'&.'&&': YVfi Ufm!'&* : YVfi Ufm	GYa]bUf	F Ya Ya VYf]b['UbX': cf[Yhj]b[
		:]Wgcb.:@i:L]bki: ":"H\Y:7`Ugg:7cibgY``cf" 'f% ++Ł	
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K YY_" ."%"A UfWX"!") A UfVX	GYa]bUf	<]g/hc/ff]WU``5a bYg]U`	
A UIVW		D`Um'; Uc'L]b[']Ub' "'"H\Y'6i g!Ghcd"' f%,' Ł	
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K YY_') . '%) 'A UfvX '! '%-	GYa]bUf	GcWJYHU`fB]głCfXYf	
A UfVX		D`Um`?ic`DUc`?ib`	
		D`Um`?ic`DUc'?ib' "'"Bc'DUf_]b['cb'CXX 8Umg"' 'f%,*Ł	
		:]`a .`3 4d[YZfWdEg Wd6Sk fK]f"`9Xk UfX`MUb[``ž%-%L	
		9ggUmDfYdUfUh]cb`ff%sı ŁXiY'Uh'h\Y'YbX'cZ'h\Y k YY_ž: f]XUm% 'A UfW\'&\$&%ž') DA	
K_YY*.^&&^A_UfVX^!^&* A_UfVX	FYUX]b[A UbXUhcfmi: "YI]"K YY_"!"Bc"WUgg	
K YY_'+.'&-'A UfVX'!'& 5df]`	GYa]bUf	⇒XYbh]hm	
		D`Um`; Uc`L]b[^]Ub` "'"H\Y`Ch\Yf`G\cfY"' f%,*E	
		:]`a . ʿE a` Y aXfZW7j [W 'fK]f" ʿ5bb ʿ< i] ž'% - \$Ł	
K YY_', .') '5df]`!'- '5df]`	GYa]bUf	8]gd`UWa Ybh	
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FYgci fWyg

DfYgWf]VYX F Ygci fWYg

5 ```fYei]fYX`fYUX]b[g`UbX`j]Yk]b[`a UhYf]U`g`fZ]`a g`UbX#cfd`UmgŁUfY`Uj U]`UV`Y`j]U`A ccX`Y"'`Ghi XYbhg`UfY YI dYWhYX`hc.`%£`\Uj Y`Wca d`YhYX`h\Y`fYUX]b[`UbX`j]Yk]b[`cZU```a UhYf]U``Zcf'h\Y`k YY_'VYZcfY`Wca]b[`hc WUggž'UbX`&Ł`Vf]b[`\UfX`Wcd]Yg`cZh\Y`fYUX]b[`a UhYf]U`g'hc`WUgg"'`6YWUi gY`mci `UfY`bchYI dYWhYX`hc di fW\UgY`UbmhYI hg`Zcf'h\]g`Wci fgYž'UbX`g]bWY`h\Y`i gY`cZU```Y`YWhfcb]WXYj]WYg`]g`dfc\]V]hYX`]b`WUggž'mci a i ghdf]bh\UfX`Wcd]Yg`cZmci f`fYUX]b[g`UbX`Vf]b[h\Ya `hc`WUgg`Zcf'ci f`gYa]bUf`X]gW gg]cb"

FYWca a YbXYX FYgci fW/g

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7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

 $H\setminus [g^*]g^*U^*bYk^*WcifgY^*^*Bc^*gh^*XYbh^*ZYYXVUW_YI]ghYX^*]b^*h^Y^k^f]h^b[^*cZh^]g^*WcifgY^*^*$

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY