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$I b hg c Z 7 f Y X h^*$

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Gi a a UfmcZh\Y 7ci fgY

7`Uggfcca 'a UbU[Ya Ybh'UbX'ghi XYbh'Yb[U[Ya Ybh]g'h\Y'bi a VYf'cbY'WcbW'fb'cZU``bYk 'hYUW\Yfg"'H\]g Wci fgY'dfcj]XYg'ghi XYbhg'k]h\'Vch\'h\Y'h\YcfYhJWD`'UbX'dfUWJWD`'Udd`]WUhJcbg'cZWUggfcca 'a UbU[Ya Ybh df]bWJd'Yg'bYWfggUfmZcf'bYk 'hYUW\Yfg'hc'a U_Y'dfcZYgg]cbU``XYWJg]cbg'k]h\'fYgdYWfhc'WUggfcca a UbU[Ya Ybh'WcbWffbg"'H\Y'ZcW g'cZh\]g'Wci fgY']g'cb'WfYUhJb['Ub'Yb[U[]b['WUggfcca 'Ybj]fcba Ybh'UbX cb'dcg]hjj Y'VY\Uj]ci f']bhYfj YbhJcbg'UbX'gi ddcfhž']bWi X]b['h\Y'Yj]XYbWY!VUgYX'dfUWJWfg'Wfi WJU`'hc YZYWMj Y'WUggfcca 'a UbU[Ya Ybh'

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8 Ya cbqffUlY 'Ub 'UV]`]lmihc 'Yb[U[Y 'gh XYbhg YZZYWf] Y`m]b 'h\Y 'YUfb]b['dfcW/gg"
- &" 8Yj Y`cd`UbX`a U]bHJ]b`U`dcq]hj Y``YUfb]b['Ybj]fcba Ybh]b`h\Y`WUqqfcca
- ' "D`Ubža UbU[Y'UbX'XY`]j Yf'dfcXi Wj Y'`Yggcbg
- ("I gY`ghfUhY[]Yg`UbX`hcc`g`hc`UXXfYgg`h\Y`X]j YfgY``YUfb]b[`bYYXg`cZgh XYbhg`UbX`a U]bhU]b``YUfb]b[Yb[U[Ya Ybh
-) " A UbU[Y:X]ZZW `hVY\Uj]ci fg:UbX:WfYUhY:U:gUZY:UbX:dfcXi Wfj Y:`YUfb]b['Ybj]fcba Ybh

5 i glfU`]Ub DfcZYgg]cbU`GlUbXUfXg Zcf HYUW\Yfg

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%"("%	8Ya cbghfuh Vfcux _bck `YX[Y'UbX'i bXYfghUbX]b['cZh\Y']a dUWhcZW `hifYz'W `hifU`]XYbh]mz'UbX``]b[i]gh]WVUW[fcibX'cb'h\YYXi WUh]cb'cZghiXYbhg'Zfca '5Vcf][]bU`'UbX'HcffYg'GhfU]h'=g`UbXYfVUW[fcibXg"	
%") "%	8Ya cbglfUHY'_bck`YX[Y'UbX'i bXYfgHUbX]b['cZglfUHY[]Yg'Zcf X]ZZYfYbhJUhJb['HYUWX]b['hc'a YYhih\Y'gdYWJZJW`YUfb]b['bYYXg'cZ ghi XYbhg'UWfcgg'h\Y'Z ```fUb[Y'cZUV]`]hJYg"	
&"*"%	±b7cfa Uhjcb′UbX′7 ca a i bjWUhjcb′HYVW.bc′c[m′	1
' "' "%	±bWiXY'U'fUb[Y'cZhYUW\]b['ghfUhY[]Yg"	I
' "+"%	8 YgWf]VY 'U'VfcUX'fUb[Y'cZghfUhY[]Yg'Zcf']bj c'j]b['dUfYbhg#WJfYfg]b'h\Y'YXi WJh]j Y'dfcWfgg"	
("%"%	⇒XYbhjZmghfUhY[]Ygʻhcʻgi ddcfh]bWi g]j Yʻghi XYbhdUfhjWjdUhjcbʻUbX Yb[U[Ya Ybh]bʻWUggfcca ʻUWfj]jhjYg"	&ž [·] '
("&"%	8 Ya cbghfUhY h\Y WUdUWhmhc cf[Ub]gY WUggfcca 'UWhj]h]Yg UbX dfcj]XY WYUf X]fYWhcbg"	1
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%":Ghi XYbhiK Y``VY]b['>ci fbU` 9bhf]Yg	&\$ı	&) #\$&#&\$&&`\$) .\$\$`DA</td><td>&ž'(ž')</td></tr><tr><td>&"'DfcV'Ya 'Gc'j]b['9 YfV]gY</td><td>(\$1</td><td>%(#\$' #&\$&&`\$) .\$\$`DA</td><td>&ž˙(ž˙)</td></tr><tr><td>' "'7`Uggfcca 'A UbU[Ya Ybh D`Ub</td><td>(\$1</td><td>&) #\$(#&\$&&`\$) .\$\$`DA</td><td>%ž'&ž'' ž'(ž')</td></tr></tbody></table>	

5ggYgga Ybh% Gh XYbhK Y``VY]b['>ci fbU` 9bhf]Yg

8i Y'XUhY. '&) #\$&#&\$&& '\$) .\$\$ DA

 $\label{eq:hug_willing} HUg_`\%! :]j Y`\&) \$!k cfX`fYgdcbgYg`hc`gdYW]Z]W`ci fbU``Ybhf]Yg`Zfca `h\Y`Z]fghih\fYY`a cXi `Yg`cZh\Y`Gh XYbh K Y``VY]b[`<i V`a cXi `Yg''`$

Gh XYbhg k]``fYWY]j Y k f]hh/b ZYYXVUW_k]h\]b % k cf_]b['XUmg cZgi Va]gg]cb"

5XX]hcbU`XYhJ]`q

Cb`]bY'AcXi `Yg'!'AcXi `Yg'%'UbX'&

Hc'a YYhž']b'dUfhž'U'bi a VYf'cZ[fUXi UhY'hYUW\Yf'ghUbXUfXgž'mci 'k]``'VY'Yl dYWfYX'hc'Wca d`YhY'h\Y'hk c cb']bY``YUfb]b['a cXi 'Yg'cb'h\Y'5i ghfU`]Ub'Gh XYbhK Y``VY]b[': fUa Yk cf_'`cWJhYX'k]h\]b'h\Y'Gh XYbh K Y``VY]b['<i V'Uh

\hdg.#\ah XYbk Y``VY]b[\i V"YXi "Ui #Xi WUhcfq#\dfcZYaq]cbU`!`YUfb]b[!\di fqYa#\di fqYHmdY1dfY!qYfi \\W

CbW/mci \uj Y`WfYUh/X`Ub`UWWti bhžmci k j``bYYX`hc`Wta d`Yh/`h\Y`&`dfY!qYfj JW/`hYUWXYf'a cXi `Yq'!

6UW_[fci bX'hc'h\Y'5i ghfu`]Ub'Gh XYbhK Y``VY]b[': fUa Yk cf_'
\hdg.##YUfb"gh XYbhk Y``VY]b[\i V"YXi "Ui #\chi fgY#]Yk "d\d3]X1' '

UbX

9b[U[]b['k]h\'h\Y'5i ghfU`]Ub'Gh XYbhK Y``VY]b[': fUa Yk cf

\hda.##YUfb"gh XYbhk Y``VY]b[\i V"YXi "Ui #Nci fqY#]Yk "d\d3]X1' (

A cXi`Y'%'HJ_Yg'UVci hicbY`\ci f'hc'Wca d`YHYž'A cXi`Y'&']g'U'V]h`cb[Yf'Ug'mci 'k]``[Yhhc'_bck 'a cfY'UVci h YUWX'cZh\Y'Z]jY'gYWf]cbg'cZh\Y': fUa Yk cf_"'Mci 'WUb'Xck b`cUX'h\Y'`ci fbU`'h\Ymgi dd`m'UbX'i gY']h'hc fYWcfX'mci f'fYgdcbgYg']Zmci '`]_Y"'5'K cfX'j Yfg]cb'cZh\Y'`ci fbU`'hYa d`UhY']g'U`gc'dfcj]XYX'Zcf'mci ']b A ccX`Y"

D`YUgY`fY[]ghYf`Uhih\Y`ghUfhcZK YY_'%cf`gccbYfž`Ug`k Y`ghfcb[`mmci 'bYYX`hc`Wca d`YhY`h\Y`&`A cXi `Yg`Vm`h\Y`ybX`cZK YY_'&"`±i]g`Ub`YI dYWfUh]cb`h\Uhmci 'k]``Wca d`YhY`h\Y`UWfj]h]Ygž'ei]nnYgž'UbX`fYUX]b[gfYWca a YbXYX`ZcfYUW\`a cXi `Y"

Mci 'k]```VY`UggYggYX`cb`Zjj Y`f) ŁfYZYWłj Y`fYgdcbgYg`hc`h\Y`a UhYf]U``Wcj YfYX`]b`h\Y`a cXi `Yg"`9UW\fYgdcbgYg`hc`h\Y`a UhYf]U``Wcj YfYX`]b`h\Y`a cXi `Yg"`9UW\fYgdcbgYg`hc`h\Y`a i ghVY`UVci h'&) \$`k cfXg`]b``Yb[h\"`Mci 'k]```VY`fYei]fYX`hc`gi Va]hmci f`Ybhf]Yg`i g]b['h\Y`K cfXXcWhYa d`UhY`gi dd`]YX`]b`A ccX`Yž'UbX`h\Yb'i d`cUX`]hhc`h\Y`Hi fb]h]b`gdUWY`]b`A ccX`Y`Vmh\Y`Xi Y`XUhY`UbXh]a Y"`H\Y`hYa d`UhY`]bWi XYg`h\Y`gh]a i`i g`ei Ygh]cbg`Zcfmci f`fYZYWł]j Y`fYgdcbgYg"

5ggYgga Ybh& DfcV`Ya Gc`j]b['9l YfW]gY

8i Y'XUhY. '%(#\$' #&\$&&'\$).\$\$'DA

HUg_'&'!'DfcV'Ya 'gc'j]b['Y| YfW]gY'!'&\$\$\$'k cfXg

Gh XYbhg k]``fYWY]j Y k f]hhYb ZYYXVUW_k]h\]b '% k cf_]b['XUmg cZgi Va]gg]cb"

5XX]h]cbU`XYhU]`g

Mci 'k]``j]Yk 'k c'WUggfcca 'a UbU[Ya YbhgWYbYg'h\Uhg\ck 'k c'j YfmX]ZYfYbhhYUW\Yfg'YghUV`]g\]b['h\Y]f Yl dYWfUhjcbgžfci hjbYgžUbX'fY`Uhjcbg\]d'k]h\ 'h\Y]f'\][\ 'gW\cc``WUggYg'cb'K YY_'%ž8Um'%'

 $\label{thm:continuous} $$ Mci_k = \colored Mci_k = \col$

5ggYgga Ybh' 7`Uggfcca 'A UbU[Ya YbhD`Ub

8i Y'XUhY.'&) #\$(#&\$&&'\$).\$\$'DA

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Gh XYbhg'k]``fYWY]j Y'k f]hhYb'ZYYXVUW_'k]h\]b'%\$'k cf_]b['XUmg'cZgi Va]gg]cb"

5XX]h]cbU`XYhJ]`g

Mci k]``WfYUhY'U'7AD'h\Uh\Ug'' dUffg'!'D\]`cgcd\mž'H\Ycfmž'UbX'DfUWfJWY''

Mci 'k]```ghUhY 'mci f'dYfgcbU``VY`]YZg`fd\]`cgcd\m\:UVci hih\Y bUhi fY`cZghi XYbha]gVY\Uj]ci f'UbX`mci f dYfgcbU``VY`]YZg`UVci hik\Uhimci f'fc`Y`cZh\Y`hYUW\Yf'Ug`WUggfcca 'a UbU[Yf'g\ci `X`VY"'Mci 'k]```h\Yb\Yf]ZmX]gW gg`k\Uhih\Ycfm#\Ycf]ghg`cf`W\bhfU``H\bYhg#_YmWcbW\dhg`fU]gYX`]b`h\Y`Wci fgY`i bXYfd]b`mci f WUggfcca 'a UbU[Ya Ybhid\]`cgcd\m\UbX`k]```[i]XY`mci f'a UbU[Ya YbhidfUW]\Wg"'Mci 'k]```h\Yb`XYHJ]`\ck mci 'k]```cf[Ub]gY`mci f`WUggfcca 'hc'dfca chY`Yb[U[Ya YbhiUbX`Uddfcdf]UhY`VY\Uj]ci fz`mci f'fi `Ygz fci h\bYgz'dfcWXi fYgz'WcbgYei YbW\gz'UddfcUW\Yg`hc``YUfbYf'X]j Yfg]hmz'UbX`fYgdcbgYg`hc'a cfY W\U``Yb[]b['VY\Uj]ci fg"'Mci 'a i ghifYZYf'hc'A ccX`Y`Zcf'h\]g`hUg_'Zcf'h\Y`hYa d`UhY`k\]W\`WUf]Z\Yg`UbX gdYW]Z\Yg'k\Uhia i ghVY`]bWi XYX"

GdYW] ZJW7f]hYf]U	"flt2fŽt

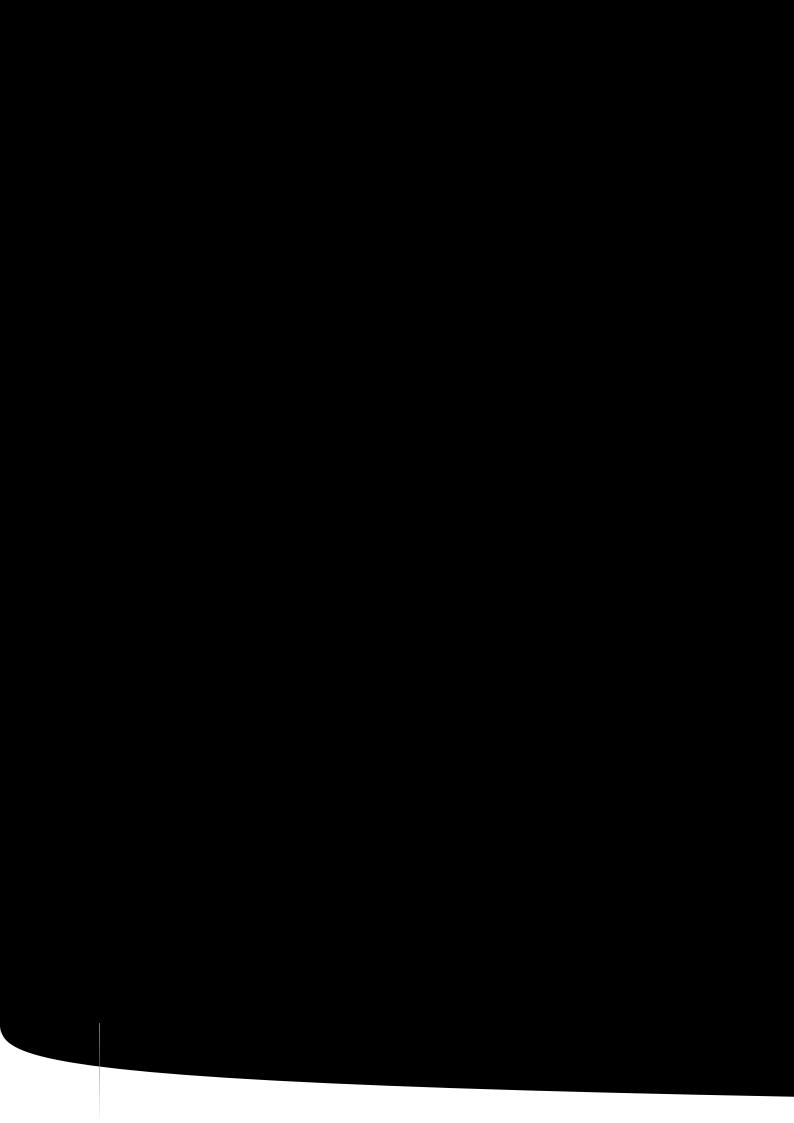
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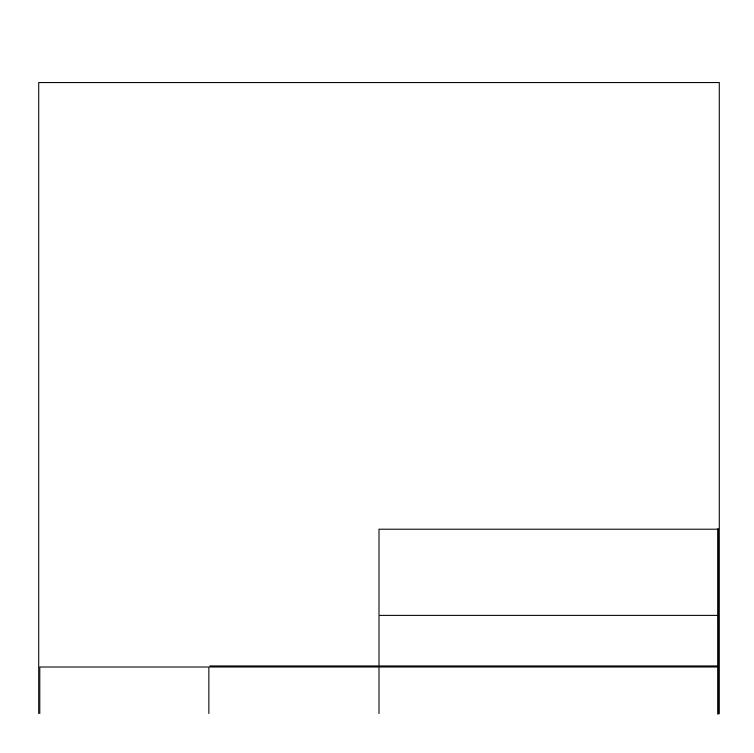
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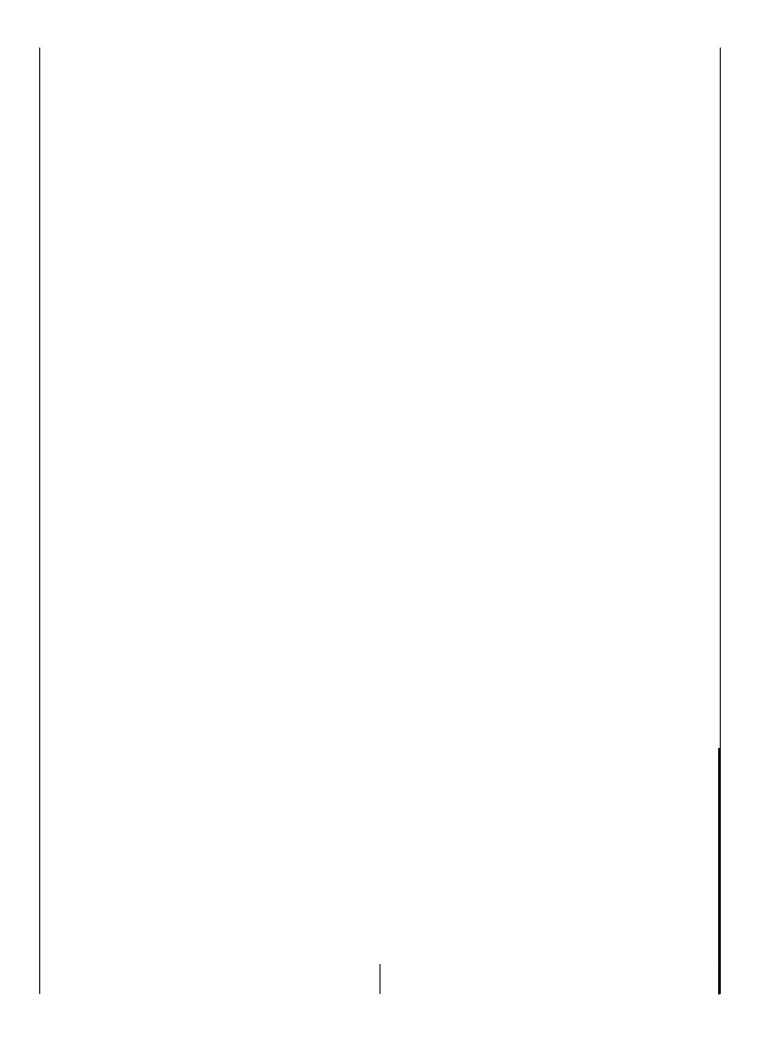
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I BGK G7<CC@C: 981 75H€B

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K YY_`%\$.`% `5df]``!`&& 5df]`	@YWhifY	Gi ddcff]b['gh XYbhg'k]h\ 'gdYW]U` 'bYYXg']b'h\Y]bWi g]j Y'WUggfcca "'K \c`Y'gW\cc`ž'hfUi a U!]bZcfa YX'UddfcUW\Yg'hc'a UbU[]b['W\U``Yb[]b[VY\Uj]ci fg"
	Hi hcf]U`	8Yj Y`cd]b['U'Wf]g]g'a UbU[Ya Ybhd`Ub" A m9I dYf]YbWr"

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

GYY'A ccX`Y'Zcf'UXX]h]cbU``fYgci fWfg'hc'Yl hYbX'mci f'_bck `YX[Y'UbX'i bXYfghUbX]b[ž'Ug'k Y``'Ug'fYgci fWfg Zcf'i gY']b'mci f'WUggfcca "

7ci fgY F YUX]b[g

HY dfYgWf]VYX hYI hZcfh]g Wci fgY]g.

9a a Yfž'9"'H"ž/ '9j Yffgcbž'7"'A "'f&\$%+L"'5]Rddc``^ ^ R_RXV^V_eWc^ZUU]V R_U YZXY dTY``] e\rangle RTYVcd'ff\sh\'YX"L"'I ddYf'GUXX'Y F]j Yfž'B>. 'DYUfgcb"

BCH9. Hc di fWkUgY hkY YVcc_ffr)) łżdYUgY WJW_cb hkY]b_\\YfY

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ"'=Imci
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a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU'`Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUhJcb"'=Imci 'Ya U]''9! hYfbU'`Gi ddcfhmci 'k]'`
Ui hca UhJWJ'`mfYWY]j Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'hc'gdYWJZJWJ'`mUg_'Zcf'cbY''
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; YbYfU``mž'UggYgga YbhiHug_g'a i gh'VY'gi Va]hhYX'Y`YWfcb]WJ``mj]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' = D`]bgHJbWYg'k \YfY'h\]g']g'bchdcgg]V'Yž']hk]``VY'gHJhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k]h\

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`=hWJb`hJ_Y`a Ubm Zcfa gžZca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

 $\begin{tabular}{ll} \hline \textbf{7.cdm}b[...] g]b[...] MY cf'j Yfmg]a] ``Uf'k cfXg'hc'h\Y'cf][] bU``HYI hcf']XYU'k]h\ci h`U\W_bck ``YX[]b[...] b[...] MY cf'i g]b[...] bU``HYI hcf']XYU'k]h\ci h`U\W_bck ``YX[]b[...] b[...] MY cf'i g]b[...] a UhYf]U`gž']XYUg'cf'WcbW'dhg'\faca '`U'Vcc_\text{z}'Ufh]WY\text{z} bU'Vcd_\text{z}'Ufh]WY\text{z} bU'Vcd_\text{z}'Ufh]WY\text{z}'Ufh]WY\text{z} bU'Vcd_\text{z}'Ufh]WY\text{$

5 WUXYa] W + DZcfa Uh]cb

8 i Y 'hc 'Yj c'j]b['UXj]\W 'VmBGK '< YU't\ ž'ghi XYbhg'a i gh'\W.Y\<u>V</u>'Zcf'i dXUh\YX']bZcfa Uh]cb 'fY[UfX]b['cb`]bY 'YUfb]b['Zcf'U``'5fhgž'8Yg][b 'UbX'5f\W.]h\Y\ni fY '\wci fgYg't\]g't\Yfa 'f]]U'A ccX`Y'cf'\wci fgY']bZcfa Uh]cb dfcj]\XYX"Ł

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY gi Va]gg]cbg [i]XY`]bYg/
- fYj]Yk cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W]Yg/
- YI Ua]bUh]cb dfcWYXi fYg/

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