

MUSC4104

 $5\,Xj\;Ub\,WY\,X\,^{\circ}G\,h\;X]Yg\,^{\circ}]b\,^{\circ}A\;i\;g]W<\,]ghc\,fm^{\prime}/\,^{\circ}7\;i\;^{\circ}h\;fY\,^{\circ}\&$

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9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

±0.5 Xj UbWYX 'Ghi X]Yg']b'A i g]W<]ghcfm/ '7 i 'hi fY'&'mci 'k]```Udd'm_bck 'YX[Y'UbX'g_]``g'XYj Y`cdYX']b Y`YWfj Ygž'WcfY 'Wci fgYg'UbX'gdYWJU`]gUhjcb 'gdYWJZJWWci fgYgž']b 'gi ghU]bYX 'Yb[U[Ya Ybhik]h\ 'U'g]b[`Y'hcd]Wźk \]W\ k]```VY'YI d`cfYX']b'XYdh\"'±0'h\Y'YUf'mghU[Yg'cZh\Y'Wci fgY'mci 'k]```ci hi]bY'Ub'YI]gh]b['VcXmcZgW\c`Ufg\]d'VmUggYa V`]b['UbX'UbbchUhjb['U'V]V`]c[fUd\mž'k \]W\ k]```Zcfa 'h\Y'VUg]g'cZU'dfYgYbhUhjcb'h\Uh]bhfcXi Wyg'mci f'hcd]Whc'h\Y'WUgg"'H\Y'WbhfU`'dUfhcZh\Y'Wci fgY'k]```VY'h\Y'Wc``UVcfUhjj Y'a Ybhcf]b['cZmci f'hcd]Wž']b'dfYdUfUhjcb'Zcf'h\Y'Z]bU`'YggUm''H\]g'a Ybhcf]b['k]```U`gc'YbUV'Y'mci 'hc'hfUbgZYf'h\Y'Wc``UVcfUhjj Y'g_]``g'h\Uhmci '\Uj Y'XYj Y`cdYX'Ug'U'a i g]WJUb'hc'gW\c`Uf'mfYgYUfW\'UbX'Wf]hJWJga "'H\fci [\h\Y'Wcbghfi Whjcb'cZU'V]V`]c[fUd\mUbX'Wc``UVcfUhjj Y'a Ybhcf]b['cZmci f'hcd]Wmci 'k]```\cbY'h\Y'Wf]hJWU`'UbX g_]``g'Zcf'k f]hJb['U'gi ghU]bYXžXYhJ]'YXž'UbX'k Y``gi ddcfhYX'Uf[i a Ybh'

7 ci fgY @YUfb]b[Ci hWca Yg

- %" I gY`UXj UbW'X`fYgYUfW\`a Yh\cXg`UbX`Uf[i a Ybh'bc`gi ddcfh'h\Y]f`fc`Yg`Ug`a i g]\W``\WfYUbcfgž gW\c`\Ufg`UbX`UXj c\WJhYg`]b`h\Y`\Wa a i b]\m'
- &" 8 fUk 'cb'Ub'YI hYbg]j Y'_bck 'YX[Y'cZfYdYfhc]fYgž'gcWjU`'UbX'\]ghcf]WU``WcbhYI hgž'UbX']XYc`c[]Yg'Ug Wf]hJWU``m'UbX'gcWjU``m'Yb[U[YX'a i q]WjUbg"
- '" k cf_`Wf]h]WJ``mk]h\`ch\YfgždfYgYbh]b[`gW\c`Ufm]XYUgʻZcfXYVUhYžX]gW gg]cbžUbX`Wca a Ybh'

HYUWX]b[GhfUhY[]Yg

H\]g`Wci fgY`dfcj]XYg`U`Zcfi a `Zcf`ghi XYbhg`hc`k cf_`k]h\`]XYUgža Yh\cXgžfYdYfhc]fY`UbX`hYW\b]ei Yg`h\Uh\Uj Y`VYYb`XYj Y`cdYX`]b`dfYj]ci g`Wci fgYg"`H\Y`ZcWj`cZh\Y`Wci fgY`UfY.`W\ccg]b[`U`hcd]WZcf`]bj Ygh][Uh]cb/Vf]b[]b[`a i g]WUbX`]XYUg`hc[Yh\Yf`]b`U`Wc\YfYbhž a Yh\cX]WU``Zcfa "

H\Y`Z|fghcZh\YgY`k]``VY`XYj Y`cdYX`]b`h\Y`YUf`mdUfhcZh\Y`Wti fgYž`Vma cXY``]b[`h\Y`gWtdY`cZdcgg]V`Y hcd]Wgž`Zcf`Yl Ua d`Y"`Gi W\X]gW gg]cbg`k]```U]a `hck UfXg`h\Y`Z|bU``Zcfa g`f[YbfYgŁ`Zcf`h\Y`Dfc`YWf`H\]g`ZcW g k]```VY`Yl hYbXYX`h\fci [\`h\Y`dfYgYbhUh]cbgž`UbX`h\fci [\`cb![c]b[`WUgg`UWf]j]h]Yg"

 $\label{thm:linear:lin$

 $\label{thm:condition} $$H\Y'h\] fX'ZcW'g'']b_g'hc[Yh\Yf'jUf]cig'UgdYWg'cZh\Y'WcifgYž']bWiX]b['h\Y'ghiXYbhdfYgYbhUf]cbgž'h\Y k f]h]b['cZXfUZgž'UbX'h\Y'Wcad]'Uf]cb#UbbchUf]cb'cZV]V']c[fUd\]Ygž'hc'WcbbYWhYUf'mWUggYg'k]h\'Z]bU'dfc'YWg''H\Y'U]a']g'hc'\Y'd'ghiXYbhg'hc'Zcfa'Wc\YfYbhdfYgYbhUf]cbgž'YUX]b['hc'h\Y]f']bU'dfc'YWh'H\Y VUg]g'cZh\]g'ZcW'g']g'h\UhghiXYbhg'k]'``\UjY'fJaY'hc'g\UfY'UbX'X]gW'gg'h\Y]f']bX]b[g'Ug'h\YmXYjY'cdž k cf_]b['Wc'`YWf]jY'mhc'XYYdYb'ibXYfgHUbX]b[g'cZaYh\cXgž'fYdYfhc]fYg'UbX'ch\Yf'fYgcifWYgž']b'U'Wf]f]WU'aUbbYf''$

H\Y`UggYgga Ybhg`gi ddcfhih\YgY`ZcW]z`UbX`dfcj]XY`cddcfhi b]hmZcf`dYYf`[i]XUbW\z`]b`UXX]h]cb`hc`[i]XUbW\UbX`ZYYXVUW_`Zfca h\Y`Wci fgY``YWhi fYf"`H\Y`gYa]bUf`Zcfa Uh\Ug`VYYb`W\cgYb'hc`cdh]a]gY`ZYI]V]`]hmcZ

hYUWX]b['UbX'`YUfb]b['dfcWYggYg"

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5bbchJhYX.V]V.]c[fUd/m	%\$ı	%#%\$#&\$&\$`\$).\$\$`DA	% <u>ž</u> &
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GhUfhXUhY. Bch5dd`]WUV`Y

eYb[h\."%\$\$\$"kcfXg"cZUbbcHJh]cbg"UbX"%\$"gcifWyg

8YHJ]`g.

 $Ghi~XYbhg'k~]```Wca~d`YhY`Ub`UbbchUhYX`V]V`]c[~fUd\backslash mcb'h\backslash Y]f'W\lambda cgYb'hcd]W'hc'VY'dfYgYbhYX`]b'k~f]hhYb'Zcfa$

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Gi Va]gg]cb'bchYg.7`Ugg'dfYgYbhUh]cb'cb', 'C WcVYf'UbX'gi Va]gg]cb'cb'A cbXUmž'&:'C WcVYf'&\$&\$'Vm) da "

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$

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8YHJ]`q.

8 fuZridfYgYbhJhjcb" 9 UWX ighi XYbhk]```[]j Y`U`gYa]bUf`dfYgYbhJhjcbg`cZU`XfUZricZh\Y]f`dfc^YWhi'5 k f]hhYb j Yfq]cb`k]```U`gc`VY`qi Va]hhYX"

a a YX]UhY`]b!WUgg`ZYYXZcfk UfX#ZYXVUW_z`UbX`bchYg`cb`gi Va]hh/X`j Yfg]cb"

5XX]h]cbU`XYhJ]`g.

9UW. 'gh XYbhk]``[]j Y'U'dfYgYbhUh]cb'ci h]b]b['h\Y]f'dfcYWhf5ggYgga YbhHUg_'' Ł'Ug'U'k cf_']b'dfc[fYgg UbX'dUfh]W]dUhY']b'h\Y'X]gW gg]cb'cZh\Y]f'k cf_"'DfYgYbhUh]cbg'k]``'hU_Y'd`UWY']b'k YY_g'+'!'%\$"'@Yb[h\ž'%\$ a]bi hYg'd`i g'%\$'!'%) 'a]bi hYg'Zcf'a Ybhcf'`YX'X]gW gg]cb'f5ggYgga YbhHUg_'(Ł''5'k f]hHYb'j Yfg]cb'cZh\Y dfYgYbhUh]cb'a i ghVY'gi Va]hHYX'hc'A ccX`Y'Vm) da 'cb'h\Y'A cbXUmVYZcfY'h\Y'dfYgYbhUh]cb']g'[]j Yb"'H\]g j Yfg]cb'a UmVY']b'dc]bhZcfa 'UbX'a Umi gY'Dck Yfdc]bhg`]XYg"'

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9ggUmcb gY`ZgY`YWYX'hcd]W!'' \$\$\$ k cfXg":]bU``UggYgga YbhiHJg_"`

9ggUmg k 1 VY a Uf YX UbX fYh fbYX k 1h ZYYXVUW

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Ghi XYbhgʻXYj]gYžfYgYUfW. ʿUbXʾdfYgYbhiUbʾYggUmh\UhYl d`cfYgʻcbYʾcfʿa cfYʿ_Ym]XYUgʻ]bʻfY`UhJcbˈhcʿa i g]W UbXʾ]hgʻgcWJUʿWbhYl h'ʿ=XYU`mžhcd]Wgʻk]``ʿUʾgcʻXfUk ʿcbˈh\YʻX]gW gg]cbgʻ]bʻk YY_gʻ%—ʿ&ʾUbXʿfY`UhYˈhcˈh\Y Zci fʻgYa]bUfʻh\Ya Ygʻ[]j Ybʻ]bʻk YY_gʻ%—ʻ) . '7 cbZ]WWʻ5ddfcdf]UhJcbžˈhfUX]hJcbžˈVcffck]b[žˈh\YZh/ 9bj]fcba YbhUbX``UbXgWJdY/UbXž@cj Y`YhW

H\Y'hcd]WW\c]WY']g'ei]hY'cdYb'Vi hgi VYWhc'Uddfcj U`Vmh\Y'7cbj Ybcf"'5g'U'[i]XYžmci 'a UmW\ccgY'hc

9UWX 'ghi XYbhifh\Y'a YbhYYŁ'\Ug'VYYb'Ugg][bYX'U'ghi XYbha Ybhcf"'5 'ghi XYbhiK =@@BCH'VY'h\Y]f'a Ybhcf'g a Ybhcf"'H\i g'YUWX 'ghi XYbhik]``'VY`cb['hc'fk c'dU]fgžcbY'Ug'U'a Ybhcf'UbX'cbY'Ug'U'a YbhYY"'H\YgY'UfY cf[Ub]gYX'Ug'[fci dg']b'A ccX'Y"'H\Y'[fci d'k]``'\Uj Y'h\Y'Zc``ck]b['bUa Y'ghfi Whi fY. 'A YbhYY'B5A 9ž A Ybhcf'B5A 9"'

H\Y'UggYggUV'Y'fYei]fYa Ybhg'cZh\Y'a Ybhcf!a YbhYY'fY`Uh]cbg\]d"'

- %" H\Y`a YbhYY`a i ghgi Va]hiU`j Yfg]cb`cZh\Y]f`8 fUZhDfYgYbhUh]cb`fbggYgga YbhHUg_`&Ł'hc`h\Y`A ccX`Y [fci d`h\Ymg\UfY`k]h\'h\Y]f`a Ybhcf'Vm) "\$\$`DA `cZh\Y`A cbXUmdf]cf`hc`h\Y`dfYgYbhUh]cb"`H\Ym g\ci `X`U`gc`gi Va]hih\Y]f`UbbchUhYX`V]V`]c[fUd\m'\5g`U`a YbhYY`mci `a i ghBC H`VY``UhYž`Ug`h\]g`k]``]a dUWhgYf]ci g`mcb`h\Y`k cf_`cZmci f`a Ybhcf"`DYbU'h]Yg`k]```VY`XYXi WhYX`Zfca `mci f`dfYgYbhUh]cb`]Z mci `UfY``UhY"`
- &" H\Y'a Ybhcf'g\ci `X'fYj]Yk 'h\Y'8 fUZhDfYgYbhUh]cb'cZh\Y]f'a YbhYYž'df]cf'hc'h\Y'dfYgYbhUh]cb"'H\Y a Ybhcf'g\ci `X'dfYgYbh'bchYgžj]U'A ccX`Yžcb'h\Y]f'fYj]Yk 'cZh\Y'A YbhYY g'k cf_ždf]cf'hc h\Y'dfYgYbhUh]cb"'H\YgY'bchYg'g\ci `X']bWi XY'dc]bhg'fU]gYX'k]h\'h\Y'a YbhYY'UbX'gi [[Ygh]cbg a UXY"'H\Ymg\ci `X'XYa cbghfUhY'h\Uhih\Y'a Ybhcf'\Ug'ZJa]']Uf]nYX'h\Ya gY'j Yg'k]h\'gca Y'cZh\Y ']hYfUhi fY'h\Y'a YbhYY'\Ug'i gYX'''
- ' "H\Y`a Ybhcf`g\ci`X``YUX`X]gW gg]cb`Uhh\Y`a YbhYY'g`dfYgYbhUh]cbž`Wca a YbhWcbghfi Whj Y`micb`h\Y dfYgYbhUh]cbž`UbX`gYY_`WUf]Z]WUh]cb`cZh\Y`a YbhYY'g`Uf[i a Ybh''

H\Y`UVcjY`fYdfYgYbhg`h\Y`UggYggUV`Y`Wca dcbYbhcZh\Y`a Ybhcf!a YbhYY`fY`Uhjcbg\]d"`6i hmci `g\ci `X WUffmh\Y`a Ybhcf!a YbhYY`fY`Uhjcbg\]d Z fh\Yf`UbX`Yb[U[Y`k]h\`X]gW gg]cb`UbX`YI W\Ub[Y`cZ]XYUg`cb`mci f hcd]W`@_Yk]gYž`]bZcfa U``ZYYXVUW_`UbX`Wcbghfi Whj Y`Wf]hJWjga `]g`YbWci fU[YX`h\fci [\ci h'

±0 'UXX]h]cbž'ZcfYUW\ 'GYa]bUf']b`k YY_g`+'!' %\$žYj Yfmgh XYbh]g`fYei]fYX`hc`\Uj Y`Yl Ua]bYX`Uh`YUghcbY`cZh\Y`gci fWg`]ghYX`]b`h\Y`UbbchUhYX`V]V`]c[fUd\mcZUh`YUghcbY`cZh\Y`ghi XYbhg`dfYgYbh]b["'H\Y`gci fW a UmVY`U`dUdYf`cf`U`vcc_`W\UdhYf`cf`U`a i g]\W`k cf_ž']b`Y]h\Yf`gWcfY`cf`fYWcfXYX`Zcfa "'Ghi XYbhg`k]``VY Ug_YX`hc`Wca a Ybhcb`h\Y`gci fW'žk]h\`dUfh]\W``Uf`fYZYfYb\W`hc`\ck 'YZZYWhj Y`mh\Y`dfYgYbh]b['ghi XYbh\\Ug i gYX`]h'

< YfY UfY gca Y ei Yghlobg ho Wobg XYf k \Yb UWhb [Ug a Ybhof.

 $= g'h Y'hcd]WWYUf'hc'mci'Ug'U'a Ybhcf3'8cYg'h Y'a YbhYY' \ Uj Y'U'WYUf'a Yh CX3''$

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5fY`h\Y`gci fWrg`i gYX`Vmh\Y`a YbhYY`UWUXYa]WcbYg3`⊋bchž`]g`h\YfY`U`fYUgcb`Zcf`h\]g3`7 Ub`mci `gi [[Ygh U`k UmcZ]a dfcj]b[`h\Y`ei U`]lmcZgci fWrg3`

 $7 \ YW \ "h Y "a YbhYYfig"] b!hYl hWhUh] cbg. "Xc"h YmgYYa "fY"] Ubhcb"g] b["Y"gci fWYg"fcf"gYWh] cbg"k]h h]b"U"g] b["Y"gci fWYb"k Yb"U"fUb[Y"cZgci fWYg"a][hVY"i gYX3"$

8 c Y g 'h\ Y 'a Y bh\ Y 'i g Y 'g Y Wc b X U f mig c i f W Y g 'k \ Y b 'd f]a U f mig c i f W Y g 'U f Y 'f Y U X] 'mi U j U] 'U Y Y 3 '

5fY`lfUbgWf]dh]cbg`UbX#cf`Ui_fU``UbU`mgYg`i_gYX3`5fY`lh\YfY`UbmigdYW]Z]Wa_Yh\cXc`c[]\W\``ei_Ygh]cbg`lh\Uh_Uf]gY3`5fY`lh\Y`dfYgYblUh]cbg`cZh\YgY`WYUf3`

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a i g]WU`dfUWf]WY'X]gWiggYX3'

⊋UbmUgdYWhcZh\Y`ghi XmgYYa gʻWcbhYbh]ci gžXcYgʻh\Y`a YbhYY`fYWc[b]gY`h\]gʻUbX`\Uj Yʻh\YmbchYX U`hYfbUh]j Yʻj]Yk dc]bhg3ʻ

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b"[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$

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K YY_'%'%('GYdhYa VYf'! % 'GYdhYa VYf	GYa]bUf	%+ 'GYdhYa VYf' -blfcXi Whcbž'd`Ubb]b['UbX'X]gW gg]cb'cZWci fgY]bWi X]b['Wci fgY'h\Ya Ygž'dfYgYbhUh]cb'hcd]Wgž a Ybhcf]b['hYUa gž'V]V`]c[fUd\]Ygž'dfYgYbhUh]cb a Yh\cXgž'ghmY'a Yh\cXgž'dfYgYbhUh]cb'XUhYg" Ghi XYbhg'UfY'Ug_YX'hc'fYUX'7\UdhYfg'%' 'cZ H\ca Ug'Hi f]bcfg'? ge[U Se E aU[S^>[WY F ZWBa'[f[Ue aXBSdf[U[bSf[a`

		H\Ya Y'' .'9bj]fcba YbhUbX``UbXgWUdY
		Ghi XYbhgʻg\ci `XʻfYUXʻ7\UdhYfʻ) ʻcZ H\ca UgʻHi f]bcfgʻ? ge[U Se E aU[S^>[XW FZWBa1f[Ue aXBSdf[U[bSf[a` f7\]WU[c. I b]j Yfg]hmicZ7\]WU[c DfYggž&\$\$, Ł"
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		H\Ya Y'(.'@cj Y'YhW
		Ghi XYbhgʻg\ci `XʻfYUX`7\UdhYf`* 'UbX`UbmfYa U]b]b[i bfYUX`gYW]cbgʻcZH\ca UgʻHi f]bcfgʻ? ge[U Se E aU[S^>[XW F ZWBa'[f[Ue aXBSdf[U[bSf[a` 'ff7\]WU[c. I b]j Yfg]hmcZ7\]WU[c'DfYggž&\$\$, Ł"
K YY_'*.'%'CWcVYf'!'&' CWcVYf	FYUX]b[&&*C WtcVYf*
		Bc'WUgg'!'fYUX]b['kYY_
K YY_'+.'&*'CWcVYf'!'' \$ CWcVYf	DfYgYblUljcb	&- 'C WrcVYf
		DfYgYbfUf]cbg
K YY_', .`&'Bcj Ya VYf'!'* Bcj Ya VYf	DfYgYblUljcb) Bcj Ya VYf
		DfYgYbhJhjcbg
K YY_''-'Bcj Ya VYf'! %'Bcj Ya VYf	DfYgYbrUrjcb	‰'Bcj Ya VYf'
		DfYgYbfUf]cbg
K YY_'%\$.'%*'Bcj Ya VYf'! &\$'Bcj Ya VYf	DfYgYbfUfJcb	% Bcj Ya VYf
		DfYgYblUljcbgžgbUW_g°

F Ygci fWyg

DfYgWf]VYX F Ygci fWYg

Hi f]bcž'H\ca Ug"'? ge[U Se E a U[S^>[\W, F Z W Ba'\f[Ue a X B S df[U[b S f[a`"'7\]\W][c.'I b]j Y fg]\mic Z 7\]\W[c'D f Y g g \&\\$, "'

FYWca a YbXYX FYgci fW/g

H < 9A 9 % 7 cbZ Wh

 $6U_Yf\tilde{z}7Uh Yf]bY"\tilde{z}4ig]WUg\tilde{z}UkYUdcb\tilde{z}2Yhbcdc\tilde{z}1hWU\tilde{z}2JhWd\tilde{z$

 $7\,fcc_{\tt Z}8\,Uj\,]X""~5\,G]I\,\,hYYbh.!\,7\,Ybh\,\,fm'7\,UhU`c[\,\,`cZDfc\,\,\,]V]hYX`A\,i\,g]W"``< agd\,\,S^aXfZW3_\,Wd[US`\,\,?\,\,ge[Ua\,^aY[US^E\,aU[Wfk]+\,^*\&.\%fGdf]b[\,\,`&\$\$-\,^LZ`M'+,\,\,^fbb`mdU[\,Yg'M'&.^UfY'fYei\,\,]fYX'fYUX]b[\,^L"'$

< U'Xi _ž'>c\b'7 "'"H]b'DUb'5``Ymcb'h\Y'A UfW. Dcdi`Uf'A i g]WžK cfX'K Uf'=ž'UbX'h\Y'E i YghZcf'U'; fYUh K Uf'Gcb[""'Babg'Sd? qe[US` V EaU[Wfkł'&*. ('fl&\$\$' Łž(-+!) %&"

C'7 cbbY`ž'>c\b'A cf[Ub"`~A i g]W]b'K UfžA i g]WZcf'DYUWY. '5'F Yj]Yk '5fh]WY"~ 7fZ` a_ge[Ua 'aYkł)) . '% fK]bhYf'&\$%%£3%&! %&+"

D]Yg`U_ž'>cbUh\Ub`F "'~Gci bX`HUf[Yhg.`A i g]W'UbX`h\Y`K Uf`]b` ≢Ue"~`>agd` S^aX? ge[Ua^aY[US^D WeW\$dJZž&*.&!' `f&\$\$+\Łž'‰' !%(-"

GUh_i bUfUlbUa ž'5\U'mU"''"GHJ[]b['K Uf. 'DYfZcfa]b['6\UfUlHJ'BUlmUa']b'7c`ca VcžGf]'@Ub_U""'5a` YdMee a` DWeWSdJZ['6S`UWZ8F>'()#%f5df]'&\$% L"'

 $\label{eq:wyfz} \mbox{Hi \underline{W}Yf\check{z}? $UfYb$`K $YghYf$\check{z}$]$Y`X""``A $i g]WK $Ufg.$`5`BYk$`7$cbZ$]$W$S"`>[fgdYkl &(.`(`f&$$-l$\check{z}' !-"` Label{eq:wyf}).$

H<9A 9 % 5ddfcdf]Uh]cbžhfUX]h]cbžVcffck]b[žh\YZh

 $5\,mYfgžHcbmYhiU^*"``Gca Yh\]b[`XUb[Yfci g`mUg_Yk `k]h\ h\ Y`k Umk YffY`hU`_]b[`UVci hfUWY`UbX`h\ Y Uffg"```E kV` Wk? ad [`Y: Wd5'Vž'>i `m, ž'&$&$"`\hdg.#k k k "ga \"Wca "Ui #W `hi fY#a cj]Yg#gca Yh\]b[! XUb[Yfci g`mUg_"""!k Umk Y!fY!hU`_]b[!UVci hfUWY!UbX!h\ Y!Uffg!&$&$$+$*!d)) - Vn"\ha`$

: U]f`Ymž`>"`f\% - &\!''A gdB 'SUWA gd? ge[Už 3Tad\Y[` S^? ge[Uł 3gefdS^{S`} Babg^Sd? ge[U [` BWdebWUf[hWHa^z \$\fint 9X]hYX`VmA UfW g`6fYYb"`7 UbVYffU. '5Vcf][]bU``Gh X]Yg`DfYggž`\% , - "`f8]ghf]Vi hYX`Vm7 Ua Vf]X[Y I b]j Yfg]\mDfYgg\t'\%+&'dd"`Babg^Sd? ge[Uł '##fl \tiz' , &!' , ("

: c`Ymž'7 Uh\Yf]bY'9"'"7 i `h fU``Hci f]ga ž'A Y]h\YU`žUbX`F Y!dfYgYbhUh]cb`cZ<Yf]hU[Y""K W\$dTaa] Xad F d\$V[f[a` \$^? ge[Už'(+'f&\$%) \\ \\ Z'%(%! \%* \$")

<]``z`>i b]dYf"\\Y`\\\Zi YbW\`cZ7cbgYfj Uhcfm: c`_`A i g]WDfc[fUa a Yg.`H\Y`G]VY`]i g`5WUXYa mi]b 7 ca dUfUh]j Y`7cbhYl h'''`7fZ` a_ ge[Ua'\aYk 8ad_

.

H<9A9". 9bj]fcba YbhUbX ``UbXgWUdY

7 c\Ybz'< Ufft'"F YdYftc]fYz'@UbXgWUdY'UbX'A Ya cfm'K]``]Ua g'g'UbX'GWi `hr'g'>ci fbYmhc'< cfgYg\cY'6YbX 7 UbHJHJ""';` fWdJg/fgc\$^? ge[U, 5 dW\$f[a` S`V;` fWdbdWf\$f[a` "'9X"'GU``mA UWJfh\i fz'6fi WY'7 fcgga Ubz F cbU`Xc'A cfY`cg"'GmXbYm'5i glfU`]Ub'A i g]W7 YblfYz'&\$\$+"

; fmggž? UfYb":"J c]WYg'cZh\Y'DUVJZ]W'h\Y'fW\&cfU`'ffUX]h]cbg'cZC WYUb]U"":FZW5S_Td[VYW5a_bS` [a` fa 5 Zad\$^? ge[Už5bXfÅ'XY'E i UXfcgžYX]hcf"''7 Ua Vf]X[Y.'7 Ua Vf]X[Y'I b]'DfYggž&\$\%\"

8]VVYbž`B]Wc`U"`~BUhi fY`UbX`BUhjcb.`BUhjcbU``⇒XYbhjlmiUbX`9bj]fcba YbhU`]ga `]b`⇒W`UbX]WDcdi `Uf`A i g]W J]XYc`UbX`A i g]W8cW a YbhUfm"~7fZ` a_ge[Ua^aYk 8aʤ_ ž% .%f&\$\$-Łž% %%) %"

±0[fUa ž8Uj]X":"A m8]fmiGffYUa". DYfY GYY[Yfž5a Yf]WUb: c`_A i g]WžUbX 9bj]fcba YbfU`DfcfYgf":Babg/Sd? ge[US`V EaU[Wfkž' %%f&\$\$, Łž&%' *"

>Ôf[YbgYbž'8c``m''''9bX`]b[ž'h\Y'Dck Yf'cZh\Y'@Ugh']b'Ub'9lh]bWh]cb!DfcbY'K cf'X'''''7`h[da`_WfS^BZ['aeabZkž'%('f&\$%*Ł"

F]WXUfXgž:]cbU""K cfXg'UbX'A i g]W-'>ci fbYmhc'<cfgYg\cY'6YbX""Ea`Ył BWdAd_S`UWS`V D[fgS^'CdYb l b]j Yfg]hmžA]hcb'? YmbYgž&\$\$, "

GWki`hnž5bXfYk"'″=XYbh]hmUbX'A Ya cfm'HYa dcfU`]hmUbX'h\Y'A i g]WcZ8Uj]X'@ a gXU]bY"″'E fgV[We [`? ge[Už&) 'f% - %±"

SSSSSS"">ci fbYmg'k]h\]b'a i g]WU``gdUWY. fYU``UbX`]a U[]bYX""@Sj ae? ge[Ua'aYk; `fWd Sf[a` S1 % fBYWYa VYf'&\$% Ł"

SSSSSS""Aig]WUbX"h\Y"AYUb]b["cZK UhYf""@Sjae?ge[Ua?aYk;`fWd Sf[a`Sf' fA Umi&\$&\$t"

 $K U`XcW\underline{z}?]a "``<agd`Wk fa: adeWeZaW4W`V F W$UZWdDWeagdJW=[f"`GmXbYmz`5i ghfU`]Ub`Aig]W7YbhfYz &$$+"$

H<9A9'(.'@cjY'YhW

8ci V`YXUmžJ Yfcb]WJ"'"Gci bXg`cZDck Yf. 5b`Cj Yfj]Yk `cZA i g]WJ``±oghfi a Ybhg`UbX ; YbXYf""'7fZ` a_ge[Ua'aYk 8adq_ž'%+.%f&\$\$, Łž''!' - "

; fYYbYž? Uh\UbbY`K "'~K ca Yb`G]b[Yf!Gcb[kf]hYfg`Ug`9IYa d`Ufm'5Wfcfg. 'H\Y`A i g]WcZF UdY`UbX 8 ca Ygh]WJ]c`YbWY"~? ge[US` V Ba ${f[Uel GiaaYf`&$\%+ž'\%&*"}$

? fUa Yfž@Uk fYbWY"`~7i `hi fY`UbX`A i g]WU``< Yfa YbYi h]Wg. `H\Y`GU`ca Y`7 ca d`YI "~`5 S_ Tq[VYWA bWdS <agd \$72&.' `f1% - \$12*&*-!&-("

A W7 `Ufmž'Gi gUb" $8W_{[[WWV[Ye, ge[U]YWVWd]S]V} = ge[U]YWVWd]S$ `V eWJ gS^{fkžA]bbYUdc`]g. `I b]j Yfg]micZA]bbYgctU DfYggž'% - %" I BGK '@|VfUfmYVcc_

 $E i Ubmž8cb"`~7 Ubcbg`]b`7c``]g]cb.'< ma bg`UbX`7cbHYa dcfUfm'7 \f]gh]Ub`A i g]W"`>[fgdYkž&(.(`f&$$-Łž' &|'-") - "`$

GWXi `mž'5bXfYk "'"'H\Y'a]bcf'ZU``ž'h\Y'a U'cf``]Zf'!!aig]Wždck Yf'UbX'h\Y'Wca dcgYf'g''V`UW_'Uffi''''9 q[X[fZ]DWh[W

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY