

MUSC3705

DYfZcfa UbWY @UVcfUhcfm7

HYfa 'H\fYY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZ:7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
>c\b'BUd]Yf	^bUd]Yf4 i bgk "YXi "Ui	: f]XUm'%\$"' \$'! %%" \$ž'cf'Vm Uddc]bha Ybhž'cf Xfcd']b	K YVghYf*%%	-',)*-)'

GWXcc`7cbHJWh =bZcfa Uh]cb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY.'f\$&E'-',)'(,)*

9a U]`.'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubgž h\Y'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fthZ''8Yg][b'7UadigYgŁUbX't\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cfWY'5WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`cWJhYX''$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'('

Gi a a UfmcZh\Y'7ci fgY

H\Y'UWei]g]h]cb'cZ\][\`miXYj Y`cdYX'Wc``UVcfUh]j Y'UbX'dfYgYbhUh]cb'g_]`g']g']bWfYUg]b[\`mi]a dcfhUbhi]b WcbhYa dcfUfma i g]Wa U_]b['UbX'h\i g'fY'Yj UbhiZcf'h\Y'dfY!dfcZYgg]cbU`'hfU]b]b['cZU```ghi XYbhg']b'U`` a U'cfg'cZh\Y'6A i g'dfc[fUa "'A I G7' +\$) `DYfZcfa UbWY @UVcfUhcfm7']g'h\Y'h\]fXž'UbX'Z]bU`ž'DYfZcfa UbWY @UVcfUhcfm7']g'h\Y'h\]fXž'UbX'Z]bU`ž'DYfZcfa UbWY @UVcfUhcfmWci fgY'Zcf'ghi XYbhg'Ybfc``YX']b'h\Y'A i g]Wc`c[mUbX'Gcb]W5fhg'a U'cfg'cZh\Y'6UW\Y`cf'cZ A i g]Wdfc[fUa "'H\]g'Wci fgY'Vi]`Xg'cb'h\Y'h\YW\b]WD'`UbX'WfYUh]j Y'Zci bXUh]cbg'XYj Y`cdYX']b'YUf]Yf DYfZcfa UbWY'@UVcfUhcfmWci fgYgž'UbX'h\Y'Uk UfYbYgg'cZU'fUb[Y'cZa i g]WD'`[YbfYg'UbX'dfUWf]WYg YbWci bhYfYX']b'ch\Yf'Wci fgYg']b'h\Y'a i g]Wdfc[fUa "'Ghi XYbhg'`YUfb'hc'Udd`mWf]h]WD'``]ghYb]b['UbX fYUX]b[g'cZdYfZcfa UbWY']hYfUhi fYž'UbX'[U]b'g_]``g'UbX'YI dYf]YbWY']b'h\Y'XYj Y`cda YbhUbX'Wi fUhcfg\]d'cZ |bbcj Uhij Y'[fci d'dYfZcfa UbWYg"

7ci fgY @YUfb]b[Ci hWca Yg

- % 5dd`miWf]hJW]ga 'UbX'UbU`mg]g`cZh\Y``]hYfUhi fY`cb`dYfZcfa UbWYž'UbX`cZa UghYf`WUgg`dfYgYbhUh]cbgž hc`h\Y`dfYdUfUh|cb`cZh\Y]f`ck b`dYfZcfa UbWY
- &" 8 Yg][b UbX dfYgYbhYZZYW]j Y YbgYa V Y a UlYf]U`UbX dYfZcfa UbWg
- ' " 5dd`mdf]bWd`Yg`cZhYUa k cf_hc`a i g]WYbgYa V`Y

HYUWX]b[GhfUhY[]Yg

j]XYc[fUd\mUbXbchYg`cb`Uh`YUgh(a UghYf!WUggYgŁ"

Dcfhzc`]cg'k]``VY'gi Va]hhYX'h\fci[\'Hifb]h]b'UbX'aUf_YX'Vmh\Y'`YWhifYf"

Gi Va]gg]cb bchYg. D`YUgY gYY XYHJ]`g`cZh\]g`Ugg][ba Ybhcb h\Y`A ccX`Y

5ggYgga Ybh'. DYfZcfa UbWY#dfYgYbHJhjcb

GHJfh:XUhY.

8 YW] g.

H\fYY']b!WUgg'dYfZcfa UbWYg'UbX'Ub']b!WUgg'WcbXi Wf]b['hYghgWUZC'X]b['UggYgga Ybh'HUg_'("

Gdc_Yb^ZYYXVUW_k]```VY^[]j Yb^Vmh\Y^`YWh fYf^]b^WUggžUbX^k f]hhYb^ZYYXVUW_g\cfhmUZhYf"

5XX]h]cbU`XYhJ]`g.

H\]g'Wca dcbYbhk]``'U`gc']bWi XY'U'g\cfhWcbXi Wfb['Yl Ua ž\Y`X']b'h\Y'ZbU``gh X]c'cZH&"'

Ghi XYbhgʻk]``XcʻUʻa]b]a i a ʻcZ%]b!WUggʻdYfZcfa UbWYjbʻH%ʻUbXʻ&ʻ]b!WUggʻdYfZcfa UbWYgʻ]bʻH' "'H\YgYdYfZcfa UbWYgʻUfYʻ[fci dʻdYfZcfa UbWYgžUgʻbchYXʻZcfʻUggYgga YbhHUg__("

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5 https://bxubwww.Fyei]fya Ybhg

D`YUgY`bchY`h\Uh`YWh fY`fYWcfX]b[g`UfY`bch'Uj U]`UV`Y`Zcf`h\]g`Wci fgY"`Gh XYbhg`UfY`ghfcb[`miYbWci fU[YX`hc UhhYbX`U```WUggYg`UbX`WcbhUWhh\Y`7ci fgY`5i h\cf]hmhc`a U_Y`U`hYfbUh]j Y`UffUb[Ya Ybhg`Zcf`WUggYg a]ggYX"

7ci fgY'GWXYXi `Y

J]Yk Wugg'ha Yhuv'y

H]a YHJV`Y

8UHY	HmdY	7 cbh/bh
C'K YY', 'GYdhYa VYf'! %%GYdhYa VYf	; fci d'K cf_	D`YUgY`UlthYbX'tc'mci f'YbgYa V`Y'Ybfc`a Ybh
K YY_'%'%('GYdhYa VYf'! %'GYdhYa VYf	Gh X]c	±xi ghfmig_]`g'X]gW gg]cb
K YY_'&.'&%GYdhYa VYf'! &)'GYdhYa VYf	Gh X]c	±bXi ghfmig_]``g'X]gW gg]cb
K YY_'' .'&, 'GYdhYa VYf'! &'C WcVYf	Gh X]c	±0Xi glfmig_]``g'X]gW gg]cb'CF 'ghi XYbhidfYgYbhUh]cbg CF 'Vch\
K YY_'(.') 'C WcVYf'!'- C WcVYf	I = = = = = = = = = = = = = = = = = = =	K_YY_g^(!) z~+!%\$^+b^WUgg^dfYgYbHJHcbg^UWWcfX]b[hc^gW.YXi^Y^UffUb[YX^Vmh\Y^YbX^cZk_YY%

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

5 WUXYa]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU`Wcbg]XYfUhjcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbz'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

.

=a U[Y 7 fYX]h

 $H\setminus]g^{-}]aU[Y^{-}]g^{-}h\setminus Y^{-}dfcdYffmicZ>c\setminus b^{-}BUd]Yfz^{-}Wc^{-}ifgY^{-}Wc^{-}b^{-}Ybcf$

7F = 7CG

7F = 7 C G 'Dfcj]XYf'7cXY. '\$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb : `W ghcX]Ubg : cZh, Y ``UbXg : cb : k \]WX : I B G K ? Ybg]b[hcb : WUa di g :]g ``cWUhYX''$