

# 7ci fgY'Cj Yfj ]Yk

### GHJZZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
@Ui_fU^7\]g`Yhh	n' ( - , &&- 4 i bgk "YXi "Ui	VmUddc]bha Ybh		
>cbYg				

### GWkcc~7cbhJWh~abZcfa Uh]cb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca ' %&ž@YjY`' žFcVYfhK YVghYf6i]`X]b[fl %(Ł

D\cbY.'f\$&E'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$ 

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg ( ' Gi a a UfmcZh\Y 7 ci fgY

# 5ggYgga Ybh

: i fh\Yf']bZcfa Uh]cb'UVci hh\Y'UggYgga YbhHUg\_g'k]``VY'[]j Yb']b'WUgg'cf'cb`]bY']b'k YY\_'%"

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
9bgYa V`Y`Wcblf]Vi ljcbž dYfZcfa UbWYgždUfhWXYW_]b[ lYgh	&\$I	Bch5dd`]WUV`Y	(
A UghYfWUgg'Wf]hjei Yg'	%) ၊	Bch5dd`]WUV`Y	1
DYfZcfa UbW\#dfYgYbHJhjcb	')	Bch5dd`]WUV`Y	%ž&
DfUWjWU``YI Ua ]bUhjcb'	' \$ı	Bch5dd`]WUV`Y	%ž&

### 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% '9bgYa V`Y'Wcblf]Vi lijcbždYfZcfa UbWgždUfh'W\YW\_]b[ 'hYgh

GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**\.'\\") 'hc'\\san ]bi h\g

#### 8YHJ]`g.

7`eW\_T^WbSdfZUZWU][Yk]``VY`ibXYffU\_Yb`]b`H%ZH&`UbX`H' "'\DX]j]XiU``mcf`]b`gaU```[fci dgz'ghi XYbhg`k]``dYfZcfa 'k cf\_g`cf`Yl WYfdhg`cZk cf\_gz'ghi X]YX`UbX`dYfZcfa YX`Xi f]b['h\Y'hf]a YghYf"'`K\YfY`Uddfcdf]UhYz Yl WYfdhg`k]``VY`gY`YWhYX`Vmih\Y'dUfh!W\YW\_]b['Yl Ua]bYf"

8]fYWhZYYXVUW\_Xi f]b[ 'fY\YUfgU`g'UbX'k cf\_g\cdg"'Gh XYbhgfWcbhf]Vi h]cb'hc'YbgYa V'Y'dYfZcfa UbWrg'UbX k cf\_g\cdg"k]``VY'UggYggYX'h\fci [\'U'dUfhW\YW]b[ 'Yl Ua ]bUh]cb"

#### 5XX]h]cbU`XYhJ]`g.

H\Y`9bgYa V`Y`7cblf]Vi h]cb`hYghik ]```cVWV f`hck UfXg`h\Y`YbX`cZUlhY\* 'Ghi X9bgYXh'ZYY'

#### 8YHJ]`g.

5 d[f[cgWek]]``VY`giVa]lthYX`UWfcgg`H%z`H&`UbX`H' "``H\YgY`k]```VY`fY`UthYX`tc`aUgthYfWUggYg`dfYgYbthYX`Vm j ]g]lt]b[ `YI dYffg`]b`U`fUb[ Y`cZaig]WU`gtm`Yg`UbX`UWfj] ]t]Yg"``7 f]t]eiY`eiYgt]cbg`k]```VY`gWUZc`XYX`tc YbWcifU[ Y`gtiXYbtg`tc`]XYbt]Zm`]ggiYg`Zcf`X]gWigg]cbz`UbX`tc`XfUk``]b\_g`Uacb[ `dYfZcfaUbW'Y`YaYbtgUbX`gWXc`Uf`m`]tYfUtifY"

 $K f] h Y b Z Y Y X V U W_k ] ```V Y d f c j ] X Y X c b `] b Y h h f c i [ \ A c c X Y ]$ 

#### 5XX]h]cbU`XYhU]`g.

cifWf]hjeiYgʻUfYʻhcʻVYʻgiVa]hhYXʻ]bʻhchU`"'GhiXYbhgʻg\ci`XʻgiVa]hUh`YUghicbYʻWf]hjeiYʻ]bʻYUWXʻcZhYfag %ž&ʻUbX'' "'H\YʻZcifh\Wf]hjeiYʻWUbʻVYʻgiVa]hhYX`UhUbmhjaYʻXif]b[`h\Y`mYUf"'Bc'acfYʻh\UbʻZcif'Wf]hjeiYg dYfʻmYUfʻk]``VYʻaUf\_YX"

 $\label{thm:condition} $$HYY'Wf]h]eiYg'aighVY'giVa]hhYX'j]U'AccX'Y'bc'`UhYf'h\Ub'cbY'kYY_'UZhYf'h\Y'aUghYfWUgg'hc'k\]W\'h\YmfY`UhY'''DYbU'h]Yg'k]``'Udd`m'Zcf'`UhY'giVa]gg]cb''$ 

Gi Va ]gg]cb bchYg.]b WUgg dfYgYbhUhjcb

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh(. DfUWh]WU Yl Ua ]bUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. '&) 'a ]bi hYg

8 YW] g.

cfdYfZcfa UbW ghi XYbhgžWj S\_ [`Sf[a`e'k]``VY'i bXYffU\_Yb']b'H' "'Ghi XYbhg'k]``dfYgYbhU'&)!a ]bi hY dfc[fUa a Y']bWi X]b[') 'a ]bi hYg'cZhYW.b]WU`k cf\_žXYj ]gYX']b'bY[ch]Uh]cb'k]h\'h\Y]f'gdYWJU`]ghhi hcfg'hc'gi ]h h\Y]f'dYfgcbU``]bhYfYghg'UbX'hfUYWhcf]Yg"''Ghi XYbhg']XYbhJZhjb['Ug'Unn'gdYWJU`]ghg'k]``dfYgYbhhYW.b]WU`k cf\_'Zfca 'h\Y'Unn'gm`UVi g'XYj ]gYX'Zcf'h\]g'Wci fgY"'H\Y'YI Ua ]bUh]cb'k]``'U`gc']bWi XY'U'g\cfhg][\h fYUX]b['hYgh'Zcf'U``ghi XYbhg"'H\Y'YI Ua ]bUh]cb'k]``'U'gc']bWi XY'U'g\cfhg][\h fYUX]b['hYgh'Zcf'U``ghi XYbhg"'H\Y'YI Ua ]bUh]cb'k]``'U'gc']bWi XY'U'g\cfhg][\h fYUX]b['hYgh'Zcf'U``ghi XYbhg"'H\Y'YI Ua ]bUh]cb'k]``'U'gc']bWi XY'U'g\cfhg][\h fYUX]b['hYgh'Zcf'U``ghi XYbhg"

: cf`Wca dcg]h]cb`gh XYbhgžbadfXa {ae`k ]```VY`gi Va ]hhYX`]b`H' "`DcfhZc`]cg`a Um`]bWi XY`Ubmbi a VYf`cZk cf\_gž Vi hh\Y`hchU``Xi fUh]cb`cZk cf\_g`g\ci `X`VY`%) !&\$`a ]bi hYg"``GWcfYg`g\ci `X`VY`gi Va ]hhYXž'U`cb[ `k ]h\fY\WcfX]b[ g`cZU```k cf\_gž'UbX`cbY`cZh\Y`fYWcfX]b[ g`a i ghVY`cZU``]j Y`dYfZcfa Ub\W''``H\]g`k ]```VY`h\Y`Z]bU` UggYgga YbhHUg\_`Zcf`Wca dcgYfg"

: YYXVUW\_, K f]hh/b fYdcfhcb dfUW]WU`YI Ua 'UbX'Wca dcg]h]cb dcfhzc`]c"

#### 5XX]h]cbU`XYhU]`g.

91 Ua 'dfc[fUa g'a i ghVY'gi Va]hhYX'Zcf'UddfcjU`j]U'h\Y'YI Ua]bUhJcb'dcftU`Vmk YY\_') "

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

# 5 hm/b XUb VW F Yei ]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$ 

# 7ci fgY GWX YXi `Y

## J]Yk 'WUgg'hja YhUV'Y

# H]a YtUV`Y

8 UhY	Hmd Y	7 cbhYbh
C 'K YY', 'GYdhYa VYf'!	Gh X]c	8cbffil&f[Yhhc'Ybfc```]b'Ub'9bgYaV`Y'Xif]b['C!
‱GYdhYa VYf		K YY_
K YY_'%'%('GYdhYa VYf'! %'GYdhYa VYf	Gi X]c	A cb'\$\$'!'%\$.'\$'Gh X]c'''(''
		K YX'\$\$'!'%\$.'\$'Gh X]c'''(
		5 hith Y 'VY[]bb]b[ 'cZh\]g'hYfa 'k Y 'k]``'VY
		X]gW gg]b[ 'YZZYW]j Y dfUWjW WW bloi Yažk Y``UVIb[ 'Zof'a i alWubažUbY A DE
		hYVX.b]ei Ygžk Y``!VY]b[ Zcfa i g]VJUbgžUbXAD5 fa i g]VVdYfZcfa UbVVUbl]YmZh\fci [\`YlUa]b]b[
		gYj YfU`W\UdhYfg`]b`h\Y`fYWcaaYbXYX`hYI h'Zcf`h\]g Wci fgYž
		VET 1912

	H\Y`Wf]hjeiY'eiYghjcb'k]``VY`dcghYX'hc'AccX`Y VYZcfY'h\Y``YWhifY"

	7`U]fY'9Xk UfXgždYfW gg]cb]ghUb) 5fh]gh]W8]fYWcfcZ9bgYa V`Y CZZgdf]b[	
	H\Y`7f]hjeiY'eiYghjcb'k]```VY'dcghYX'hc'AccX`Y VYZcfY'h\Y``YWhifY"'7f]hjeiYg'UfY'XiY']b'kYY_'%\$'cb :f]XUm'	
K YY_'%\$.'%*'Bcj Ya VYf'! &\$'Bcj Ya VYf	A cb#K YX`ghi X]c`WUggYg`Ug`dYf`mci f`ck b`h]a YhUV`Y" Bcb!UggYggYX`]b!WUgg`dYfZcfa UbWYg"`D`YUgY`Vf]b[	

### FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

K ]``]Ua cbž5'&\$\$( 'f9X"Ł? ge[US^7j UW'W UW, efdSfWY[We S` V fWUZ` [cgWe fa W ZS` UWbWdYad\_ S` UWC | ZcfX | b]j Yfg]miDfYggžC | ZcfX

#### FYWca a YbXYX FYgci fWYg

 $6 \ UWX \ Z'7 \ "'D"'9"'fW+)' \ E'7 \ eeSk \ a` fZWF \ dyW3 \ d'aXB'Sk[`Y = WkTaSdV;`efdy_ W' fe'HfUbg''K ]``]Ua'A ] \ HWXY``'' BYk' \ Mcf_. 'K "Bcfhcb' UbX'7 c"'% (-$ 

6Yfa Ubž6":f&\$\$\$L":@afWe Xta\_ fZWb[S` [efe TW UZ":BYk '< Uj Yb.:MUY:I b]j Yfg]lmiDfYgg"

6YfbUWZD"f% +\* L"FZW[`fWbdWfSf[a` aX8dW UZ ea`Y"'@cbXcb.'; c``UbWn"

6i h½'>"ž/ '8 fYm½ gž'@''f9Xg"Ł"'f&\$\$%Ł" B 'Sk[`Yi [fZ: [efack, FZWZ[efac[US^SbbcaSUZ fa? ge[US^BWbAc\_S`UW'7 Ua Vf]X[Y.'7 Ua Vf]X[Y'I b]j Yfg]hmDfYgg"

7 cY\`cžJ "'fPX"Ł"'f&\$\$' Ł"'5 S\_ TqVYWUa\_ bS` [a` fa fZWYg[fSd"7 Ua Vf]X[ Y. 7 Ua Vf]X[ Y'I b]j Yfg]lmiDfYgg"

7 c ZZ]b ž 6 " 'f 16 \* \$! & 2 " F Z We [ Y Wde dWb Wdfa [dW

 $< i \ Xgcb\check{z}F "`f\% - + \&"`E fa `W f[_W fZWZ[efa ck a XfW_ ba \ dgTSfa"`7 \ Ua \ Vf]X[Y.`7 \ Ua \ Vf]X[Y$ 

### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY