

MDIA5023

Di V`]WF Y`Uh]cbg'GhfUhY[m

HYfa 'H\fYY''##'&\$&\$

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7 fYX]hDc]bhg *

Gi a a UfmcZh\Y'7ci fgY

H\]g`Wci fgY`k]``dfcj]XY`mci `k]h\`U`WcbhYa dcfUfm'UbX`Wf]h]WU``i bXYfghUbX]b[`cZh\Y`Z bXUa YbhU``WcbW'dhg UbX`dfcWfggYg`h\Uhi bXYfd]b`di V`]WfY`Uh]cbg`ghfUhY[]Yg`UbX`WJa dU][bg"`H\Y`Wci fgY`ZcWi gYg`cb`ghfUhY[]W Wca a i b]WUh]cb`d`Ubb]b[ždi V`]WfY`Uh]cbg`fYgYUfW\žh\Y`fc`Y`cZWi`hi fY`]b`di V`]WfY`Uh]cbgž]ggi Yg`UbX`Wf]g]g Wca a i b]WUh]cbž`WcfdcfUhY`gc\Y]U``fYgdcbg]V]`]hm'UbX`Yh\]Wg"

7ci fgY @YUfb]b['Ci hWca Yg

- %" → XYbhZmu fub[Y cZuddfcuWkYg ubx XYVuhYg uvci hhkY buh fYžfc Y ubx j u i Y cZdi V j WfY uhcbq
- &" =XYbhjZmUbX`X]ghjb[i]g\`VYfk YYb`ghfUhY[]WUbX`hUWjWU``]bg][\hg`]b`h\Y`WcbhYl hg`cZd`Ubb]b[`UbX WUa dU][b`XYj Y`cda Ybh
- ' " 5dd`miWca a i b]WUh]cb`UbX`glfUhY[miWcbWldhg`hc`WUa dU][b`d`Ubb]b['UbX`dfYdUfY`di V`]WfY`Uh]cbg Wca a i b]WUh]cb`HYI hg`UbX`a UhYf]U`q
- ("5dd`miU`fUb[Y`cZYggYbh]U``hU_YUk Umg`Zfca `Yh\]WU``di V`]WfY`Uh]cbg`dfUWf)WY`UbX`WcfdcfUhY`gcW]U`fYqdcbg]V]`]hm

HYUWX]b[GhfUhY[]Yg

H\]g'Wci fgY'k]``Ya d`cmgYa]bUfg'hc'dfcj]XY'fY[i `Uf'cddcfhi b]hJYg'Zcf'dfcXi Wfj Y'X]U`c[i Yg'UbX Wc``UVcfUhJcb"'Ghi XYbhg'k]``Yb[U[Y']b'ghfUhY[mXYj Y`cda Ybh'UbX'UbU`mg]gž'X]gW gg'WcbhYa dcfUfm gw\bUf]cgz'UbU`mgY'WgY'ghi X]Yg'UbX'XYj Y`cd'Wca a i b]\WhJcb'hYl hg"'H\Y'`YUfb]b['dfc\Wgg'k]``Ya d\Ug]gY 'fYU`'k cf'X''di V`]\WfY`UhJcbg'dfcV'Ya gž'W.U``Yb[Yg'UbX'cddcfhi b]hJYg'hc'Ugg]ghighi XYbhg'hc fY`UhY'h\YcfYhJ\U'\Wb\Wdfy'hc'h\Y'dfU\Wf]\W'cZdi V`]\WfY`UhJcbg'':\DhYfU\Wf] Y'UbX'fYZY\Wf] Y'hYU\X]b['UddfcU\XYg'k]``VY df]cf]hJgYX'U`cb['k]h\'Xb\Wdhi U``ZfUa Yk cf_g'UbX'Udd`]YX'a Yh\cXg''

5ggYgga Ybh

A cfY^[i]XY^]bYg^UbX^gi ddcfhfYgci fWg^UfY^Uj U]`UV^Y^cb^A ccX^Y"

5ggYgga Ybh'HUg_g'

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7 Ua dU][b·7 f]h]ei Y·) \$1	&&#%#&\$&\$`%%) - `DA</td><td>%ž&Ž Ž(</td></tr></tbody></table>	

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5XX]h]cbU`XYhJ]`g.

A cfY^[i]XY^]bYg^UbX^gi ddcfhfYgci fVWg^UfY^Uj U]`UV^Y^cb^A ccX^Y"

5ggYgga Ybh& Hi hcf]U`ZUWj`]hUhjcb

GhUfh'XUhY.

@Yb[h\. '%\$ a]bg dYf dYfgcb]b U [fci d

8YHJ]`g.

; fci d'Ugg][ba Ybh''% a]bg'dYf'dYfgcb"

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 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$

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Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh]g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb g]a]'Uf]hmfYdcfhg"

5 hm/b XUb WY F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$

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	K_YY_`&.`&%}G&dhYah&\f\f'! &)`GYdhYa_VYf	GYa]bUf	F YgYUfVX `]b`Di V`]WF Y`UI]cbg
Ŋ	K. YKgY_'7.c&,)GOFdhYFa HocVYFYF'! &`C. WhoCVYf	GYa]bUfF Y`Uhjcbg	Di V`]WFY`Uh]cbgž7caa %b]WUh]cbAcXY`g`UbX DYfgi Ug]cb
D	KV`]WW <u>F</u> `Y``Ùjh]Cb\gneor\calaib]\	Whjcb A cXY`gʻUbX	

Di V`]W

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DfYgWf]VYX F Ygci fWYg

5j U]`UV`Y`cb`A ccX`Y"

FYWca a YbXYX FYgci fWYg

5j U]`UV`Y`cb'A ccX`Y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\]g`Wci fgY`k]```i bXYf[c'Wcbh]bi U``XYj Y`cda Ybhzij]U'ZYYXVUW_'dfcWYggYg'gi W\`Ug`ghi XYbhigi fj Ymg`UbX Wc``Y[]U``fYj]Yk"'K Y`h\YfYZcfY`hU_Y`mci f`ZYYXVUW_j YfmgYf]ci g`m''K Y`k]```U`gc'Wc``YWh]bZcfa U``ZYYXVUW_]b`h\Y`Zcfa `cZUbcbma ci g`gi [[Ygh]cbg'UbX`Wca a Ybhg`h\fci [\ci hih\Y`Wci fgY"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$