

ARTS3124

7 c``UVcfUhjj Y'DYfZcfa UbWY'A U_]b[

HYfa 'H\fYY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
H\Yfcb [.] GW\a]Xh	h'gW.a]Xh4 i bgk "YXi "Ui	VmUddc]bha Ybh	K YVghYf"‱%	-',)"&%

GWXcc~7cbhJWh~abZcfa Uhlcb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY.'f\$&E'-',)'(,)*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$

5ggYgga Ybh

 $H\Y'Wci fgY'k]```fYgi `h']b`U'Z]bU`'dYfZcfa UbWY'ci fWca YžVi hh\Y'Zcfa Uh'h\]g'k]```fU_Y'k]```cVj]ci g`miXYdYbX cb'h\Y'g]h Uh]cb'fY[UfX]b['7 C J =8!% 'UbX'fYglf]Wf]cbg'cb'd\mg]WJ``[Uh\Yf]b["'H\Y'dYfZcfa UbWV ci fWca Y'k]```hU_Y'h\Y'gUZYhmcZU```dUfh]WfdUbhg']bhc'UWVci bhž'UbX'Wci `X']bWi XY'Ui X]YbWYg'k \c'UfY``]a]hYX]b'bi a VYfZ'k UhW\]b['cb`]bYž'UbX#cf'dfYgYbh'Vi h'd\mg]WJ``m'X]ghUbWYXž'hU_]b[']bgd]fUh]cb'Zfca 'Ufh]ghg'k \c\Uj Y'k cf_YX'WfYUh]j Y`mk]h\'fYa chYbYgg'UbX'dUfh]WfdUh]cbž'Ug'k Y``'Ug'XYj]g]b['ci f'ck b'gc'i h]cbg''$

5hdfYgYbhžmci 'g\ci `X'_YYd'mci f'gW\YXi `Y'Wca d`YhY`mWYUf'Zcf'h\Y'Zc``ck]b['h]a Yg']b'k YY_'%

- 5 HYWkb]WU``F Y\YUfgU``k]h\ mci f'[fci d'Xi f]b['h\Y'XUmcZA cbXUmž'&' 'Bcj Ya VYf''
- 5 '8 fYgg 'F Y\YUfgU``Zfca ')!- da 'cb'Hi YgXUmž'&('Bcj Ya VYf" 'fMci 'a UmbYYX'hc'a YYhk]h\ 'mci f [fci d'Xi f]b['h\Y'XUm'Ug'k Y``"Ł
- Di V`]WdYf7cfa UbWg`UbX`dcgHg\ck `fY!gYh7fca `)!%\$`da `K YXbYgXUm': f]XUmž'&)!&+ Bcj Ya VYf"

5ggYgga Ybh'& '7`Ugg'Wf]h]ei Yg

GhUfh XUhY.

8YHJ]`g.

7 f]h]WU``Wca dUf]gcb`cZhk c`dYfZcfa UbWYg`Vmimci f`dYYfgž`]b`fY`Uh]cb`hc`Wci fgY`h\Ya Ygž'fYUX]b[gž'UbX fYZYfYbWYX`Ufh]ghg"`%&) \$!%) \$\$`k cfXg"

Hi fb]h]b Wca a Ybhg"

.

5ggYgga Ybh' .; fci d'dYfZcfa UbW

GhUfh'XUhY.

8YHJ]`g.

:]bU``[fci d`dYfZcfa UbWY`C&) ı `[fci d`a Uf_`Ž`&) ı `]bX]j]Xi U``Wcblf]Vi h]cb"Q

.

5XX]h]cbU`XYHJ]`g.

Di V`]WdYfZcfa UbWY`]b'k YY_'%%

Hi fb]hb gYhb[. H\]g]g bchU Hi fb]hb Ugg][ba Ybh

5 hyb XUb WY F Yei] fYa Ybhg

 $H\Y'Wci fgY'fi bg'cb'U'd\]`cgcd\mh\Uh'Wcbh]bi U``ma U_]b['`WfYUh]j Y'k cf_ž'k UhW.]b['ch\Yfgž'UbX'[]j]b['`UbX fYWY]j]b['`ZYYXVUW_']g'YggYbh]U``hc'h\Y'`YUfb]b['dfcWYgg''$

5g'U'dfUWfW!VUgYX'Wći fgYžh\]g'Wći fgY']g'dfYgYbhmgW\YXi `YX'hc'fi b'ZUW!!hc!ZUW']b'H' '&\$&\$"`'5`` hYUW\]b['k]`hU_Y'd`UW']b'h\Y'h\YUhfY'gdUW'g'cZh\Y'9ga Y'H]a VYfm7fYUh]j Y'DfUWfW'@UVžk\YfY'h\YfY']g gi ZJWJYbhgdUW'hc'a U]bhU]b'd\mg]WU`X]ghUbW'Zfca 'YUW\'ch\Yf'i bXYf'W ffYbh[cj Yfba Ybh[i]XY`]bYg"'K Y k]``'U`gc'i gY'VfYU_ci hgh X]cg'gc'h\UhYUW\'[fci d'\Ug']hg'ck b'gdUW'Xi f]b['h\Y'gYWcbX\\U`ZcZh\Y'Wci fgY"

₹ħ\Y`Wċi fgY`fi bg`ZJWY!rc!ZJWY`Ug`d`UbbYXž'k Y`k]```\Uj Y`łk c`a YYf]b[g'dYf`k YY_. 'U'(!\ci f`[i]XYX k cf_g\cd`fĭGh X]c "Łžk \YfY`h\Y`Wċi fgY`Wċbj YbYf`k]```ZJW]]lUhY`dfUWf]WJ``Y! YfW]gYg`UbX`]bhfcXi WY`\]ghcf]WJ` UbX`WċbhYa dcfUfmY! Ua d`Yg/UbX`U''!\ci f`dfUWf]WJ``g\Uf]b['fi'C h\Yf″Ł'k\YfY`mci 'k]```g\UfY`UbX`fYWY]j Y ZYXVUW_'cb`k cf_'mci '\Uj Y`XYj Y`cdYX`]bX]j]Xi U``mfk YY_g'\(\frac{1}{2}\)(Ł'UbX`]b`[fci dg`fk YY_g')!\%\$Ł"`

₹WjfW a ghubWg fYei]fY h\unn\Y Wci fgY a cj Y dufhju``mcf'YbhjfY`mcb`]bYž'u`hYfbuhj Y hYuW\]b['UbX`g\uf]b[uffub[Ya Ybhg`k]``VY di h`]b'd`UW\ž'k]h\ a cfY Ya d\ug]g cb'UgmbW\fcbci g'hUg_g']b'cfXYf'hc'a]b]a]gY cb`]bY ZJhj[i Y"

K \Yth\Yf`ZJWY!tc!ZJWY:cf`cb`]bYž]b`th\YgY'dfUWf]WJ``gYgg]cbg`mci `k]```UWf]j Y`mYb[U[Y`k]th\WcfY'Wci fgY WcbhYbhth\Uhk]``YbUV`Y`mci `tc`XYa cbglfUthY`UltU]ba YbhcZfcf`\Y`d`mci `tc`UltU]b£'7 @C '%ff'5dd`m Uddfcdf]UthY`th\YcfYt]WJ``UbX`d\mg]WJ``fYgYUfW\`dfUWf]Wfg`]b`th\Y`WfYUt]cb`cZdYfZcfa UbWf`£\/7 @C '& ff'9j U`i UthY`UbX`Wcbglfi Wf]j Y`mWf]t]ei Y`a UthYf]U``[YbYfUthYX`]bX]j]Xi U``mUbX`]b`[fci dg`Ug`dUfhcZth\Y`WfYUt]j Y dfcWfgg^*£\UbX`7 @C '' `ff'K cf_'YZYWf]j Y`m]b`ga U```[fci dgžXYj Y`cd]b[`Wc``UVcfUt]j Y`g_]``g`UbX`gY`ZfY`]UbW]b`d`Ubb]b[`UbX`YI YW t]cb*£"

5 thY b X U b WY 'U h U `` 'W U g Y g ']g 'a U b X U h c fm]b 'h]g 'W i fg Y " I b Y I W g Y X 'U V g Y b WY 'Z fca 'a c f Y 'h U b '&\$ i 'c Z W U g Q Y g 'k]`` f Y g i `h]b 'h Y 'U k U f X 'U 'Z J]` [f U X Y "

7 ci fqY GWX YXi `Y

J]Yk 'WUqq'Ha YHJV'Y

H]a YHJV`Y

8UHY	HmdY	7 cbhYbh
K YY_'%'%('GYdhYa VYf'! %'GYdhYa VYf		⊕hfcXiWh]cb*hc*Wc*`UVcfUh]jY*aU_]b[.*GYY]b[*h\Y a]fUW`cig
K YY_'&.'&%GYdhYa VYf'! &)'GYdhYa VYf		Hi YgXUmgh X]c.'5i hcV]c[fUd\]WU``dYfZcfa UbWY`UbX]X]cgmbWfUh]W[Yghi fYg" : f]XUmg\Uf]b[.'Gc`c'HUg_'%fUi hcV]c[fUd\]WU``a UdŁ dfYgYbhYX`]b!WUgg
K YY_'' .'&, 'GYdhYa VYf'! &'C WcVYf		

K YY_^).^%&^CWcVYf^!^%* CWcVYf		6Y[]bb]b['[fci d'k cf_
K YY_'*.'%-'CWvcVYf'!'&' CWvcVYf		: `YI 'k YY'gY`Z[i]XYX'[fcid'YIYfW]gY'hc'VY Wcad`YhYX']bXYdYbXYbhm
K YY_"+."&*"C WcVYf"!" \$		[fci d'k cf_'Ž'ZYYXVUW_
C WcVYf	5ggYgga Ybh	7 `UggʻWf]hjei Yʻfl&) \$!%) \$\$ʻk cfXgŁ'Xi Yʻ) da ʻcb A cbXUmž'&* ʻC WcVYf
K YY_', .'&'Bcj Ya VYf'!'* Bcj Ya VYf		[fci d'k cf_'Ž'ZYYXVUW_
K YY_''- 'Bcj Ya VYf'! %' 'Bcj Ya VYf		[fci d'k cf_'Ž'ZYYXVUW_
K_YY_'%\$.'%*'Bcj Ya VYf'!		[fci d'k cf_'Ž'ZYYXVUW_
&\$`Bcj Ya VYf		K YY_'%% 'DfcXi Wfjcb'k YY'HYW\b]WU`fi b'cb A cbXUm'XfYgg'fY\YUfgU`cb'Hi YgXUm'di V`]W dYfZcfa UbWfg'cb'K YXbYgXUmž'H\i fgXUmž'UbX : f]XUmfH67'VUgYX'cb'fYgH]Wfjcbg'cb'[Uh\Yf]b[Ł

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

H\YfY`k]```VY`g\cfhVfYU_g`Uh`gi]hUV`Y`dc]bhg`Xi f]b[`YUW\`k YY_`mWUgg"`I gY`cZd\cbYg`UbX``Udhcdg`k]`` bchVY`dYfa]hhYX`Xi f]b[`hc`YUW\`WUgg.`VchhY`cZk UhYf`UbX`U`gbUW_ž`Wca ZcfhUV`Y`Wch\Yg`ZcfacjYaYbhž`UbX`U`bchYVcc_`UbX`dYb"

FYWca a YbXYX FYgci fWYg

 $D`YUgY`gYY`@Y[Ubhc`Zcf'h\Y``]ghcZfYWcaaYbXYX`hYIhg`fY`UhYX`hc`YUW\`kYY_fg'h\YaYž'k\]W\`k]``VY`igYZ\`Zcf'h\Y`WUgg`Wf]h]eiYg''$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bz'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbz'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfz'Vi hjZmci 'hy'Yd\cby'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/