

# **ARTS2033**

DcYlfmiUbX DcYlfJWg

HYfa 'H\fYY''##'&\$&\$

## 7ci fgY'Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
GYUb Dfmcf	g"dfmcf4 i bgk "YXi "Ui	6mUddc]bha Ybh	FcVYfh KYVghYf'&%	-',)+'%)

#### GWkcc~7cbhJWh~abZcfa Uh]cb

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Fcca ' %&ž@YjY`' žFcVYfhK YVghYf6i]`X]b[fl %(Ł

D\cbY. f\$&£'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVq]hY. k k k "Ufhq"i bak "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$ 

### 7ci fgY 8YW] g

7 fYX]hDc]bhg \*

Gi a a UfmcZh\Y'7ci fgY

Gi VYWh5fYU. 7 Y /[eZ

 $H\g^*WcifgY^*WUb^*U^*gc^*VY^*ghiX]YX^*]b^*h^Y^*Zc^*ck]b[^*gdYW]U^*]gUh]cbg.^5dMSf[hWI_dff]^Y$ 

#### 7cifgY@YUfb]b[CihWcaYg

%" 8Ya cbglfUlY 'U'\_bck `YX[Y'cZdcYlfmik f]ltYb']b'U'j Uf]YlmicZd`UWYg'Uli'U'j Uf]YlmicZlt]a Yg" &" @cWUlY 'dcYa g']b'l\Y]f'\]gltcf]WU'ž'gcW]U'ž'W \h fU'ž'UbX'YWcbca ]WWcblYI ltq"

' " 9I d`U]b`a U'cf`]ggi Yg`]b`h\Y`h\YcfmicZdcYlfmžUbX`Vf]b[ 'h\Uhih\Ycfmihc`VYUf`cb`h\Y`UbU`mg]g`cZ gdYW]Z]WdcYa g"

("H\]b\_Wf]hWU``mUbX`k f]hY`dYfqi Uq]j Y`mUVci hdcYlfm"

### HYUW ]b[ 'GhfUhY[ ]Yg'

H\]g`Wci fgY`]bj c`j Yg`k YY\_`m`YWi fYg`UbX`hi hcf]U`g"``H\Y``YWi fYg`UfY`XYg][ bYX`hc`XYa cbghfUhY`h\Y`WcgY UbU`mg]g`cZdcYhfmžhc`dfcj ]XY`ghi XYbhg`k ]h\`\]ghcf]WU``VUW\_[ fci bX`hc`h\Y`gYhdcYa gž'UbX`hc`Yl Ua ]bY a U^cf`Yl Ua d`Yg`cZh\Y`h\YcfmcZdcYhfm"

@YWNifYg`acXY``Zcf`ghiXYbhg`h\Y`kUmg`]b`k\]W\`]bZcfaUh]cb`]g`cf[Ub]gYX`UbX`Uf[iaYbhg`UfY`aUXY`]b`h\YX]gW]d`]bY`cZ9b[`]g\"`Hihcf]U`g`fYei]fY`ghiXYbhg`hc`Yb[U[Y`UWh]jY`mk]h\`h\Y`fYUX]b[g`UbX`hc`XYjY`cd`h\Y]fckb`Wf]h]WU``fYgdcbgYg"``H\]g`cWWfg`h\fci[\`ghiWifYX`WUgg`X]gWigg]cb`UbX`gaU``![fcid`YIYfW]gYg"

## 5ggYgga Ybh

F YZYfYbWJb[ "Ghi XYbhg UfY ZYY hc WkccgY k \]WkYj YfifYZYfYbWJb[ igmghYa hkYmdfYZYf" < ck Yj Yfz]b a U\_]b[ ihk]g Wkc]WY ghi XYbhg g\ci `X VYUf ]b a ]bX hkY di fdcgY cZfYZYfYbWJb[. hc [ i ]XY hkY fYUXYf hc gci fW gz UbX k \Yb bYW ggUfmhc hkY fY Yj UbhdUfhcZh Y gci fW ]b ei Ygh]cb" 5 b Ui hkcflXUhY gmghYa k ]hkci hdU[ Y bi a VYfg ]g hki g dYfZYWhmUddfcdf]UhY Zcf gWkc`Ufmk f]hjb[ `XYU`]b[ ik ]hk g\cfhk cf\_g XY`]j Yf]b[ 'g]b[ `Y ]XYUgž Vi hhkY gUa Y gmghYa ]g ]bUddfcdf]UhY k \Yb WJhjb[ 'U bcj Y ž'U dcYa ž'U gi VghUbhJU` ci fbU`UfhJW ž'U gWkc`Ufma cbc[ fUd\žcf'U k cf\_cZUbm`Yb[ hk UbX Wca d`Yl ]hm' \D hkcgY WUgYgž'dU[ Y cf `]bY bi a VYfg UfY YggYbhJU` Zcf'hkY fYZYfYbWY hc Xc ]hg ]bhY`YWNi U`k cf\_"

 $\textbf{K cfX'7ci bhg''5```ei chUh]cbg``UbX`fYZYfYbWYg``UfY`]bWi XYX`]b``k cfX``Wci bhgZ'k \]`Y``V]V`]c[fUd\]Yg``UfY`bch''5 a Uf[]b``cZ\%i \`UVcj Y``UbX`VY`ck \`h\Y`gh]di ``UhYX`k cfX``Wci bhk ]```VY`UWW'dhYX`k ]h\ci hdYbU`hm''$ 

#### 5ggYgga YbhHUg\_g

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A Uʻcfʻ9ggUm	) \$1	&&#%\$#&\$&\$`%%) - `DA</td><td>(</td></tr><tr><td>HU_Y!<ca Y^91 Ua ·</td><td>) \$1</td><td>&' #%%#&\$&\$`%%) - `DA</td><td>%ž(</td></tr></tbody></table>	

#### 5ggYgga Ybh8YhJ]`g

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GhJfh'XUhY.

8YW]`q.

&ž\$\$\$ k cfXgž ZYYXVUW dfcj ]XYX j ]U Hi fb]h]b

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 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

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&\$\$\$ k cfXg": YYXVUW\_dfcj ]XYX j ]U Hi fb]h]b"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg XYbhg XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg$ 

## 5 https://bxubwww.Fyei]fYaYbhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`m'YbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhfY'fY'WcfX]b[g''' } \\$ 

## 7ci fgY'GWXYXi `Y

#### J ]Yk 'WUgg'h]a YhUV'Y

## H]a YtUV`Y

8 UhY	Hmd Y	7 cbh/bh
K YY_"%"%("GYdhYa VYf"!	@YWhifY	H\Y'=XYU'cZDcYlfm
% 'GYdhYa VYf	@YWhifY	J c]VW
K YY_'&.'&%GYdhYa VYf'! &)'GYdhYa VYf	@YWn fY	J c]W/
K YY_" .'&, 'GYdhYa VYf'! &'C WcVYf	@YWn fY	J ]g]cb
K YY_'(.') 'C WcVYf'!'- C WcVYf	@YWn fY	A YIfY
K YY_")."%&"CWcVYf"!"%* CWcVYf	@YWn fY	A i g]W
K YY_'*.'% 'C WcVYf'!'&' C WcVYf	@YWn fY	: @9L 'K 99?
K YY_'+.'&*'C WcVYf'!'' \$ C WcVYf	@YWn fY	HfUbg`Uhcb
K YY_', .`&Bcj Ya VYf'!`* Bcj Ya VYf	@YWn fY	< ]ghcfm
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K YY_'%\$.'%*'Bcj Ya VYf'! &\$'Bcj Ya VYf	@YWhi fY	H\Y` <del>_</del> XYU'cZDcYlfm'F Ydf]gY

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

FYWca a YbXYX FYgci fWYg

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

Gh XYbhg k ] ``VY YbWc i fU [ YX hc '] ``c i hcb`] bY A m9 I dYf] YbW gi fj Ymg 'Uhh Y YbX cZh Y Wc i fgY gc 'h Uh Z fh Yf ] a dfcj Ya Ybhg a Um VY a UXY "

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) "''' %cf'Ya U] hya 'cb'Yl hy'fbu'hy'ngi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9| hy'fbu''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Yl hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9| hy'fbu''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hy'Yd\cby'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

### 5 WUXYa ]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua ]bUhcb dfcWXi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybh fY/
- gh XYbhYei ]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uh]cb#dfchcWc`g![i ]XY`]bYg#

⇒a U[Y'7fYX]h

flŁ

7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;

#### 5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb : `W ghcX]Ubg : cZh, Y ``UbXg : cb : k \ ]WX : I B G K ? Ybg]b[hcb : WUa di g : ]g ``cWUhYX''$