

# **ARTS2020**

7 fYUhjj Y K f]hjb[

HYfa "H\fYY" ## "&\$&\$

## 7ci fgY'Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWJhjcb	D∖cbY
FcUbbU'; cbgU'j Yg	fcUbbU4 i bgk "YXi "Ui	Vm'Uddc]bha Ybh		

#### GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i ]`X]b[ ff %( Ł

D\cbY. f\$&Ł'-',) '(,) \*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$ 

## 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

H\Y'U]a 'cZh\]g'Wci fgY']g'hc'ZJa]]Uf]gY'mci 'k]h\'U'k]XY'fUb[Y'cZWcbhYa dcfUfm`]hYfUhi fY'UbX'hc YbWci fU[Y'mci 'hc'Yl dYf]a Ybhk]h\'U'j Uf]YhmcZX]ZZYfYbhghm`Yg'UbX'[YbfYg']b'mci f'ck b'k f]h]b["'6m i bXYfhU\_]b['h\]g'Wci fgY'mci 'k]``UWei]fY'U'dfUW]WU'i bXYfghUbX]b['cZ\ck']hYfUhi fY'k cf\_g'VmXYj Y`cd]b[  $g\_]``g`cZWf]f]WU``UddfU]gU`ž'YX]f]b[``UbX`fYXfUZf]b[$ 

# 5ggYgga Ybh

%L'A ]bcf'5gg][ ba Ybh ' FYUX]b[ 'Ug'U'K f]hYf '

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 			1
1	1		1
<u> </u>			1
1			l
-			I
 <del> </del> '			1
1			l
<u> </u>			1

 $\label{thm:linear} YghUV`]g\ 'U'Wf]hJWU``cf`ch\ Yfk\ ]gY`hcbY/\ck \ ']g'Yj\ YfmXUm``Ub[i\ U[Y'gYa\ UbhJWU``mfYcf]YbhYX'k\ Yb\ ]bWi\ XYX`]b'U'dcYa\ /\ck \ 'XcYg'U'`]bY'VfYU_'g\ ]Zhicf`a\ i\ `hJd`mh\Y`a\ YUb]b[\ 'cZU'gYbhYbWf3$ 

- 7 ca dYhYbWY`]b`WfUZhg\_]``g`ffY`Uh]b['hc`bUffUh]jY'jc]WYž'd`chghfi WhifYž``]bY`VfYU\_gž'X]U`c[iYž'YhWŁXYjY`cdYX'h\fci[\cihhY'gYaYghYfž'UbX'fY`YjUbhhc'h\Y'[YbfY'k]h\]b'k\]W\'h\Y'kcf\_'cdYfUhYgfdcYhfmž'Z]WhcWf]h]W]ga ž'YhWŁ
- =a dfcj Ya YbhcZk cf\_'Zfca 'XfUZha Ubi gWf]dhk cf\_g\cddYX']b 'WUgg'hc'ZbU''dc`]g\YX'd]YW

## 5 https://bxubww.Fyei]fyaybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$ 

## 7ci fgY'GWXYXi `Y

### J]Yk 'WUgg'hja YhUV'Y

## H]a YtUV`Y

8 UHY	HmdY	7 c bhYbh
C'K YY', 'GYdhYa VYf'!		
%%GYdhYa VYf   K YY_'%'%('GYdhYa VYf'!   % 'GYdhYa VYf		⇒hfcXiWh]cb. D`chij Yfgi gʻ7\UfUWhYf3
74 OTATILA VII		
		DWSV[`Ye ShS['ST Wh[S ? aaV W
		8UjY'9[[Yfgž'"BchYgʻZcf`UʻGhcfmcZUʻAUbʻK\cʻK]`` Bch8]Yʻ5`cbY″
		<y`yb`; td="" ufbyfž'~@]zy`cz5fh~<=""></y`yb`;>
		6UfVUfU <sup>*</sup> K Y <sup>*</sup> gž <sup>*</sup> "; cf[Yci g"
		< Y`Yb`: ]Y`X]b[žcdYb]b[ 'hc'4q[VYWf <a` 6[sck<="" td="" we=""></a`>
		DYHYf 7 UfYmž"8 c Mci @cj Y A Y3"
K YY_'&.'&%GYdhYa VYf'! &)'GYdhYa VYf		A mGhcfmcf"H\Y]f"Ghcfm3": ]fghdYfgcb"UbX"H\]fX! dYfgcb"BUffUh]cb
		DWSV[Ye ShS['STWh[S?aaVW
		DUMF]W_Gig_]bXžcdYb]b[~hc~BWMg_W

	6fcbkmbʻ@YUž"H\Y`Ch\Yf′KUmiCih"
	FcVYfh; fUmžï@UhY: Yffmï
	7 cfU`` <i '8="" ``ž´″<ck="" c'8="" td="" u_y'@cj="" y3″<="" yg'a="" yhywhj=""></i>
	GhYd\Yb`@Uk fYbWYž"⊕j'h\]gʻDcYhfm3"
	GUfU\` <c``ubx`6ultiz"h\u`uggc[fud\m"< td=""></c``ubx`6ultiz"h\u`uggc[fud\m"<>
	< YfU'@bXgYmi6]fXž'"? YUhg']g'8YUX'gc': i W_'a Y'Zfca 6Y\]bX"
	6Yh\"G\Yd\YfXž <b>"</b> : f]XUm'A UfW\"&%'&\$\$' <b>"</b>
	A ]WXUY`6Uh]b]W <b>z"</b> 8YUh\^]b′5i hab"
	<b>5bXfYk `NUk UW_]ž"%</b> " '"ž? Sec'g₩ <b>5</b> VW

1	; `YbXU`5XUa gž'″F YWcbglfi Wf]cb`cZUb`9j Ybh″
	A Uf[ UfYhi5lk ccXž'cdYb]b[ 'hc'FZW: S`V_S[Vie FS^W
	6fYh9Ughcb'9``]gž'"Acfb]b['''Zfca'3_Wd[US`BekUZa
	=gUVY``Y`5``YbXYž' <b>"</b> D\Ubhca `DU`UW <b>Y"</b>
	F Uma cbX'7 Ufj Yfž'"5fY'Mci 'U'8cWcf3"
	@Ui fYbh'6]bYhžcdYb]b[ 'hc': : Z:
K YY_', .`&'Bcj Ya VYf'!'* Bcj Ya VYf	DfchYgh'DcYhfm'UbX'l fVUb DcYhfm
	DWSV[Ye ShS[STWh[S?aaVW
	5Xf]YbbY F ]W\z`"F UdY"
	9f]WU'8 Uk gcbž"HcXUm'='Hc`X`8 cbU`X`Hfi a d"
	; ][ 'F mUbž'″-Z='\UX'U'; i b″
	CimUb['M ž":iW_'Mci'5ighfU`]Už"'"5'FUW]gh 7\]bYgY':Uh\Yf"
	5b]U'K U'k ]Whž'"5i glfU']U"
	@Yg`AiffUmž"H\Y`GigdYbg]cb`cZ?bcW_"
	DUi ``8 Uk gcbž'″H\Ub_gʻZcfʻth\YʻDcYa gž'DUi `]bY < Ubgcb″
	@]cbY`: c[Ufhmž'": iW_'U```8YdUfha Ybhg"ž'"5]bffiBc 5Vc'K UmicZ7caaib]WUh]cb"
	? YbbYh\ 'G`Yggcfž' "K ]``]Ua 'GlfYYh"
	7 cfU`' <i ``ž'"@j="" th="" yfdcc`"<=""></i>
	>c\b`A UhYYfž″A i g`]a `K ca Ubž'7 Ufhcb″
	>]```>cbYgž"GUhifXUmAcfb]b[`]b`5g\Z]Y`X"
	Ca Uf AigUž″75D +15@@9HH9FG″
K YY_''- 'Bcj Ya VYf'! % 'Bcj Ya VYf	$H\Y``mf]W = `UbX`]hg`Ch\Yfg`$

	·
	DWSV[ Ye ShS[STWh[S ? aaVW
	5bbY'GYI hcbž'"H\Y'5XX]WM'/"H\Y'6U``UX'cZh\Y @cbY`m'A Ughi fVUhcf"
	G\Ufcb'C`Xgž"K\Yb`]h7caYg"
	Fid]'? Uifž"UddUfYbhm]h]g'ib[fUW/ZicZaY"/"]kUbhhc'Udc'c[]nY'hc'U``h\Y'kcaYb"
	6fcbk mb "@YUž""8 c[ "8 Umg"/" "5 "D `UWV"/"6fcbk mb @YU"
	7 cfU`` <i "<="" ``ž'"gduffck="" td=""></i>
	CimUb['M ž"HfUbg`Uh]b['A mgY`Z'
	6cV'DYfY`a Ubž'″7\fcb]WA YUb]b[ g″
	6YfbUfX'K Y`hz'"='GhcddYX'K f]h]b[ 'DcYhfm'
	9a a U'6UfbYg'"=Ua 'U'A Ub"
	H"7" Hc`VYfhž'"A mA Y`]ggU"
K YY_'%\$.'%*'Bcj Ya VYf'! &\$'Bcj Ya VYf	: fca Z]fghXfUZhhc df]bhYX dU[Y. XfUZh]b[ž fYj ]g]cbžUbX YX]h]b[

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

 $\label{eq:K-YY-mfYUX} $$ K YY_mfYUX] $$ [g'k]^: VY'UjU] UJ'UV'Y'j] U'A ccX'Y $$$ 

F YWca a YbXYX F Ygci fWYg 7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

#### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

### 5 WUXYa ]W±bZcfa Uh]cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUh]cb dfcWYXi fYg/
- gdYWJU``Wcbg]XYfUljcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uh]cb#dfchcWc`g![i ]XY`]bYg#

.

#### =a U[ Y 7 fYX]h

D\chc VmA UI '6 O\a Y 'cb 'I bgd`Ug\

#### 7F = 7CG

7F = 7 C G 'Dfcj ]XYf'7cXY. '\$\$\$-,;

#### 5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $K Y "UW\underline{b} ck" YX[Y"h Y"6YXY[U"dYcd"Y"k \c"UfY"h Y"ffUX]h]cbU"W ghcX]Ubg"cZh Y"UbXg"cb"k \] WX "I BGK"$