

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g

#### 7 cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
>Ua YgʻK ][[]bg	0 0	d\cbY'cf'j]XYc'WU``"	Fcca " ‰ž "YjY`" FcVYfh KYVghYf 6i]"X]b[	

#### GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY.'f\$&E'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7i ghcX]Ubgž h\Y'6YXY[U'`fPYbg]b[hcb'WJa di gŁž'; UX][U'`fP]hmUbX'5fh\''8Yg][b'7Ua di gYgŁ'UbX'h\Y'B[i bbUk U` dYcd'Y'fbi ghfU`]Ub'8YZYbW': cfW'5WJXYa m]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJa di g'cZl BGK ']g`cWJhYX''$ 

### 7ci fgY 8YHJ] g

### 7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y 7ci fgY

5hUbm[]j Yb'a ca Ybhzcb'Ubm[]j Yb'XUmzk Y'UfY'VY]b['gc`X'gca Yh\]b["'=ra UmVY'Ug'g]a d`Y'Ug'U'dfcXi Wh cf'Ug'Wca d`Yl 'Ug'Ub'YbhJfY'k UmcZ`]ZY"'K Y'WU``h\]g'ImdY'cZWca a i b]WUhJcb'fdfca chJcbfl'5bXz``]\_Y']hcf'bchz ]hg'hYbXf]`g'UfY'dfYgYbh]b'Yj YfmZUWhcZci f'`]j Yg"'Cb'U'[fUbi `Uf'`Yj Y`z'h\YgY'hYbXf]`g'a Um]bZi YbW'h\Y Wch\Yg'k Y'k YUf'cf'h\Y'ybhYfhU]ba Ybhk Y'j ]Yk "'Cb'U'a cfY'[fUbX'gWUYz'dfca chJcb']g'WYbhfU`'hc'ci f dc`]hJwj'UbX'ci f'YWcbca m'

K Y``]j Y`]b`U`dfca chjcbU``W`hi fY"`H\]g`Wti fgY`k]```dfcj]XY`mci `k]h\h\Y`\]ghcfmUbX`h\Ycf]Yg`cZ\ck `h\]g W`hi fY`WJa Y`hc`VY"`5bXž`Ug`h\Y`hk c`a cghg][ b]ZJWJbhUbX`YghUV`]g\YX`ZJY`Xg`k]h\]b`dfca chjcbž'di V`]W fY`Uhjcbg`UbX`UXj Yfhjg]b[ `k]```dfcj]XY`h\Y`ZcW g`Zcf`Xc]b[ `h\]g"`Gcž'k\Yh\Yf`mci `d`Ub`hc`VY`U`dfUWjhjcbYfž'U gW.c`Ufž'cf`g]a d`mU'a cfY`]bZcfa YX`WjhjnYbž'h\]g`Wti fgY`cZYfg'j]hU``\_bck`YX[ Y`cb`\ck `dfca chjcbU` Wta a i b]WJhjcbg`cdYfUhY`UbX`h\Y`fYdYfW gg]cbg`h\Ym\Uj Y`\UX`Zcf`ci f`gcWJYhm'``

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

%" 5dd`miWcbWidng`UbX`h\Ycf]Yg`cZdfca ch]cb'hc`Wf]h]WU``miUbU`mgY`dfca ch]cbU``hYI hg"
&" 8YgWf]VY`UbX`X]gW gg`h\Y`\]ghcfmcZdfca ch]cbU``dfUWf]WYg`UbX`h\Y]f`]a dUWficb`gcW]Yhm"
' " 7 fUZhU`WYUfz`Wf]h]WU``UbX`dYfgi Ug]j Y`Uf[ i a Ybhk \]W\`UXXfYggYg`h\Y`\]ghcf]Yg`UbX`W``hi fYg`cZ
dfca ch|cb"`

### HYUWX]b[ GhfUhY[]Yg

±bh/fuwij YždufijwijdulicfmubXfYZYwiji Yfh/uwXjb[fuddfcuwXygik]``VYfdf]cf]hjnYXfucb[fk]h. Webw/dhi U`Zfua Ykcf\_gfubXfudd`]YXfa Yh\cXg"fh\ygYfh/uwXjb[fufluh/[jYgYbweifu[Yighi XYbhgihci]a a YfgYfh\ya gY`j Ygjbrh\Yf\]ghcfmubXfwfjhjwu``UddfcuwXygicZdi V`]wfY`UhjcbgfubXfuXj Yfhjg]b[fi]bfcfXYffhcfdfca chYffYZYwijcbfubXh\Yfudd`]wuhjcbfcZh\jaa UhYf]u``]bfvch\ZhiffYuwuxya [wubXi]bXi qhfmdi fqi ]hg"``

# 5ggYgga Ybh

### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
FYZYWjcb'YIYfWjgY'	&\$I	\$&#\$+#&\$&%`\$, .\$\$`DA</td><td>%ž''</td></tr><tr><td>8]gWigg]cb:@YUXYfg\]d:</td><td>' \$1</td><td>5g'UffUb[YX'k]h\'Hihcf"</td><td>&ž''</td></tr><tr><td>7 f]h]WU`9ggUm</td><td>) \$1</td><td>\$*#\$,#&\$&%`\$,.\$\$`DA</td><td>%ž'&ž''</td></tr></tbody></table>	

## 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh'% FYZYWhjcb Yl YfWjgY

8 YHJ]`g.

fWUgY`ghi Xm'z`%\$\$\$!&\$\$\$`k cfX`WUgY`ghi Xm'': YYXVUW\_j ]U`Wci fgY`@A G"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh& 8]gW gg]cb @YUXYfg\]d

8YHJ]`g.

f\( \bar{b}\) X]j ]Xi U``ma Uf\_YX\( \bar{b}\) a ]bi h\( \g \) Xi f]b[ \( \bar{U}\) gd\( \bar{W}\) \( \g \) U'\dUff\) \( \bar{b}\) \( \b

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh' . 7f]h]WU`9ggUm

8 YHJ] g.

ffYgYUfW. Wca dcbYblt:Uddfcl "&\$\$\$ k cfXg"": YYXVUW\_dfcj ]XYX"j ]U @A G"

	Į.	

# Fygci fWyg

DfYgWf]VYX F Ygci fWYg

B#5

FYWca a YbXYX FYgci fWYg

8 Uj ]gž'5 Yfc b" %\$% "'Bda\_ af[a` S^5 g fgdWe, FZWD [eWS` V E bdWsV a X3 VhWdf[e[` Ył BgT fU D WSf[a` eł ? Sd] Wf[` Y S` V 4 ds` V[` Yž'7 Ua Vf]X[ Y. 'Dc`]lmiDfYgg"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhjb['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bz'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'j UbhXcW a Ybhg\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k ]'`

JbWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\wUh]cb"'\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k ]'`

Ui hca Uh]\wU'`mfYW']j Y'U'h]\w\_Yhbi a VYf\mathred{Image: hY'Yd\cbY'mci 'k ]\tag{Y}\wu''mug\_'\mathred{Image: Wa'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY'Yd\cbY''mci 'k ]\tag{Y}\wu''mug\_'\mathred{Image: hY'\mathred{Image: hY'\mathred{I

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5 WUXYa ]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[ hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg '[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua ]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/
- gh XYbhYei ]hmUbX'X]gUV]`]hm/

•