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- &" 7 f]h]WU``mUbU`mqY``]hYfUfmUbX'j]qi U``hYl hq`hc``cWUhY'h\Y'q][b]Z]WUbWY'UbX'cdYfUh]cb'cZh\YqY']qqi Yq
- '"=XYbrjZmrh\YrfY`Urjcbg\]d'VYrk YYbrhYl hg`UbXrh\Y]f`gcVjU``UbX'W`ri fU``WcbhYl hg
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H\Y`FYUX]b[``UbX`FYgYUfW\``@c[``]g`XYg][bYX`hc`gi ddcfhdf]a UfmWti fgY`fYUX]b[``UbX`dfcj]XY`U`gWUZC`X]b[Zcf'k cf_'cb'h\Y`a U^cf'YggUm''K Y`k]```X]gWi gg'h\]g`UggYgga Ybh]b`h\Y`Z]fghgYa]bUfg"'A ccX`Y`k]```]bWi XY]bZcfa Uh]cb'i gYZ\``Zcf'WcgY`fYUX]b[`dfUWf]WY`UbX`h\Y`ghfi Wfi fY`cZUbbchUhYX`V]V`]c[fUd\]WYbHf]Yg"
Ghi XYbhg`fYdcfhih\Y`VYbYZ]hcZh\Y`Wta d`Yh]cb'cZU`_YmUggYgga YbhVYZcfY`h\Y`hYfa `UggYgga Ybh`cUX]bWYUgYg'hc`Z\```WUdUV]m"

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8 YW] g.

 $\label{thm:conditional} $$H^g'a U^cf'YggUmfYei]fYg'mci 'hc'dfcXi WY'U'dUdYf'cZ&ž) $$k cfXg'fY`Uh]b['hc'a UhYf]U``Zfca 'h\Y'gYWcbX\U'ZcZh\Y'Wci fgY''$

5XX]h]cbU`XYhU]`g.

H\]g`YggUmfYei]fYg`h\Y`gh XYbhhc`i gY`hk c`hYl hg`Zfca `Zfca `k YY_g`)!%\$`cZh\Y`Wci fgY"`

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh']g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb q]a]`Uf]hmfYdcfhg"

5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

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K YY_'%" %A Um!"(">i bY	@YWi fY	#alfcXi Whjcb'hc'5F HG' \$(\$'UbX'hc'h\Y'Z Y'X'cZ dcghWc'cb]U```]hYfUfmWf]hJW]ga ž'ZYUhi f]b['lk c dcYhg.''5`]'7cVVm9W_Yfa Ubb'fgY`YWhJcbg'Zfca 'DgTk ? aa` (YZffi'UbX'gca Y'5`VYfhK YbXhfJ` e[VWGe fZW 6WSV fž f9`]nUVYh\'A WA U\cb'UbX':]cbU'A cff]gcbŁ
	GYa]bUf	GY`YWYX`UgdYWg`cZdcgłWc`cb]U```]hYfUfmWf]h]W]ga. 5`]`7`cVVm9W_Yfa UbbžgY`YW]cbg`Zfca 'DgTk ? aa` (YZffdcYlfmŁ'5`VYfhK YbXlfz;`e[VWGefZW 6WSV fdcYlfmŁ
K YY_'&.'+'>i bY'!'\% >i bY	@YWhi fY	#Y`UbX`]b`9i fcdY.`dcYa g`cZGYUa i g` <yubymž 9Uj Ub`6c`UbXžBi U`U`B]`8\ca\bU]```f9`]nUVYh\ A WA U\cb`UbX`:]cbU`A cff]gcbŁ</yubymž
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	GYa]bUf	F YUggYa V`]b['h\Y'ZfU[a Ybhg. 'A]W\UY` C bXUUhYfi3` [1e 9 Zaef
K YY_').'&, '>i bY'!'&'>i `m	@YWi fY	=g`UbXg`Ug`]XYU``Wc`cb]Yg. '>Ua U]WU'?]bWU]Xž'3 E_S^ B'SUW/g\cfhbcb! Z]WI]cbŁ'UbX`DcYhfmcZh\Y 7 Uf]VVYUb. ']bWi X]b['gY`YWYX'dcYa g'cZ8YfY_ K U`Wchtz'C`]j Y'GYb]cfz'? Ua Ui '6fUh\k U]hY'fUj U]`UV`Y cb'A ccX`YŁ'f9`]nUVYh\'A WA U\cbŁ
	GYa]bUf	=g`UbXg'Ug']XYU``Wc`cb]Yg. '>Ua U]WU'?]bWU]Xž'3 E_S^B'SUW/g\cfhbcb! Z]Wf]cbŁ'UbX'DcYlfmcZh\Y 7 Uf]VVYUb. ']bWi X]b['gY`YWWYX'dcYa g'cZ8YfY_ K U`Wchtž'C`]j Y'GYb]cfž'? Ua Ui '6fUh\k U]hY'fUj U]`UV`Y cb'A ccX`YŁ'
K YY_'*.') '>i `m'!'- '>i `m	F YUX]b[F 958-B; 'K 99?
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K YY_'+.''&\'>i `m'!''*'`>i `m	@YWhifY	K f]h]b['< YfY 'UbX'H\YfY.'7\]a Ua UbXUfg'5X]W\Yfg' FZWFZ[Y 3dag`V Kagd@WU] 'fI]cbU'A cff]gcbŁ
	GYa]bUf	K f]h]b[' <yfy'ubx'h\yfy.'7\]a td="" ua="" ubxufg'5x]w\yfg'<=""></yfy'ubx'h\yfy.'7\]a>

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	GYa]bUf	Hcif]ga 'cb'h\Y'fff Ygff'H\caUg'?]b[fgiFcgfZS`V 4d[YZfI SfWd
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	GYa]bUf	Jc]WYž[Ygh fY'UbX'fYg]ghUbWY.'Gdc_Yb'KcfX DcYhfmcZ5i ghfU']UžBYk'NYU'UbX'UbX'h\Y'DUW]Z]W GY`]bU'Hig]hU'U'AUfg\ž'?Uh\m>Yhb]`!?]']bYfž'Hig]UhU 5j]Už'AY`Ub]Y'Aibib[[iff!K]``]Uagž'AY`Ub]Y'FYY'' Cb`]bY'fYgcifWygž'`]b_g'cb'AccX`Y
K YY_'%\$.'&'5i [i gh!'* 5i [i gh	@YWhi fY	Hci [\'Wca YXm'? SqW? g`]ZSqSł7hWdkEWUdWf FZ[`Y fg\cfhZJWf]cbŁ5B8''4akłX]fYWfYX'VmHU]_U KU]fjhjfZjaŁff9`]nUVYh\'A WAU\cbŁ
	GYa]bUf	? Sq[W?g`]ZSqSt7hWdkEWUdWfFZ[Yfg\cfhZ]Wf]cbb5B84aktX]fYWfYX'VmHU]_U'KU]hjhj'fZ[at

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DfYgWf]VYX F Ygci fWYg

 $\label{thm:conditional} H\Y'5FHG'\ \$(\$^&\$\&\%A\ ccX`Y'g]hY']bWi\ XYg'U'hYI\ h)ghZcf'h\]g'Wci\ fgYZ'UbX'h\Y'fY`UhYX'gd'e\ gi\ dd`]YX'Vmh\Y I BGK '6cc_g\cd''H\YgY'Wb'VY'cfXYfYX'Zcf'XY`]j\Yfmcf'd]\W_!i\ d'Zfca\ 'h\]g'gi\ dd`]Yf'''Ghi\ XYbhg'Wbb'UWei\]fYfYUgcbUV'Y'YX]h]cbg'cZh\Y'hYI\ hg'Zfca\ 'gci\ fWYg'Wcbj\Yb]Ybhhc'h\Ya\ "'H\Y'Z]`a\ 'k\]``VY'UWWYgg]V'Y'j\]U'h\Y I BGK\ ``]VfUfm''$

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FYWca a YbXYX FYgci fW/g

 $H\Y'A\ ccX'Y'g]hY'Zcf'h\Y''Wci\ fgY']bWi\ XYg'U'V]V']c[\ fUd\m'Zcf'5F\ HG'\ \$(\ \$'Ug'k\ Y``'Ug'a\ UhYf]U'g'cb'@Y[\ Ubhc''\ 5bmh\]b['Wca\ di\ `gcfmk\]``'VY']bWi\ XYX'cb'h\Y'A\ ccX'Y'g]hY'Ug'U'Wcbj\ Yb]YbW''Zcf'ghi\ XYbhg'''$

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

Gh XYbhZYXVUW_`]g`]a dcfhUbhZcf`h\Y`YZYWfj Y`XYg][b`UbX`]a d`Ya YbhUh]cb`cZh\]g`Wci fgY"`H\Y UggYgga YbhZcf`5F HG' \$(\$`\Ug`VYYb`fYXi WYX`]b`fY`Uh]cb`rc`gh XYbhZYXVUW_`UVci hhf]a YghYf`cUXg"`K Y``cc_`Zcfk UfX`hc`Z fh\Yf`ZYYXVUW_`f]bZcfa U``UbX`Zcfa U`Ł'h\fci [\ci hh\]g`Wci fgY`UbX j]U`A m9I dYf]YbW'`Uhh\Y`YbX`cZHYfa `&"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
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; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc']W]Yg/
- YI Ua]bUh]cb dfcWYXi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX'X]gUV]`]hm/