

ARTS2091

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H\]g`Wci fgY`]g`XYg][bYX`rc`Ugg]ghghi XYbhg`]b`WcbW\dhi U`]n]b['th\]g`a ca YbhcZfUX]WU``fYWcbZ[[i fUh]cb'' A cj]b['Uk UmZca 'hfUX]h]cbU``ghUh]Wa cXY`g`cZa Ugg!a YX]U'rck UfXg`h\Ycf]nUh]cbg`Ya d\Ug]n]b['a cV]`]mz WcbbYWhj]mubX`dYfgcbU`]nUh]cbz`h\]g`Wci fgY`]g`XYg][bYX`rc`[]j Y`h\Y`ghi XYbhg`h\Y`i d!rc!XUhY i bXYfghUbX]b['cZWcbhYa dcfUfma YX]U`h\Uhh\Ymk]```bYYX`rc`VYWca Y`gi WWYggZ ``a YX]U`dfUWh]h]cbYfg"'H\]g Wci fgY`XcYg`bchdf]a Uf]`mZcW g`cb`gdYW]Z]Wa cV]`Y`hYW\bc`c[]Yg`fk \]W\`h\Ya gY`j Yg`k]```VY`ci ha cXYX g\cfhintz`Vi hfUh\Yf`gYY_g`rc`dfcj]XY`Ub`Uddfcdf]UhY`UbX`fY`Yj UbhUWUXYa]WZFUa Yk cf_`Zcf`ghi XYbhg`rc Yb[U[Y`k]h\`UbX`XYj Y`cd"'H\Y`UggYgga Ybhg`]b`h\]g`Wci fgY`UfY`XYg][bYX`rc`dfca chY`U`XYYd`Yb[U[Ya Ybh k]h\`h\Y`_Ymh\YcfYh]WU``WcbWYdhg`UbX`rh\Ycf]ghg`Ug`k Y```Ug`rc`YbWci fU[Y`gY`ZfYZYI]j Y`UbX`Wc``UVcfUh]j Y``YUfb]b["`

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'j UbhXcW a Ybhg\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k]'`

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