

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D\cbY
H\Yfcb [·] GW\a]Xh	9 -	Uddc]bha Ybh	‰%∵@YjY``%ž FcVYfh KYVghYf	-',)"&%

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&L'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. 'k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd_S`UWEfgV[We

 $H\ [g'Wc'\ fgY'WUb'U'gc'VY'ghi\ X]YX']b'h\ Y'Zc``ck\ [b[\ 'gdYW]U']gUh]cbg.\ 5\ dWSf[hWI\ \ d[f[\ Yl\ 6\ S`\ UWE\ fgV[Wel\ ?\ WW[Sl\ 5\ g'fgdWS`\ V\ FWUZ`\ a'aYk$

DYFZcfa UbWY]g th Y dfuwljw th UhrVf]b[g wYYUhjj]mhc `]ZY" #i]g Z bXUa Ybhu` hc th YUhfY!a U_]b[žXUbWY UbX `]j Y Ufh'hc ghcfm hY ``]b[]b Z]a žUb]a Uhjcb UbX [Ua Yg/UbX hc Wa a i b]wUhjb[']XYUg]b di V`]WfY`Uhjcbg UbX Vi g]bYgg" K \Yh Yf mci f ZcW g]g Ufh]gh]WYbXYUj ci f]b dfcZygg]cbU` dfcXi WhjcbžW `hi fU` dUfh]W]dUhjcb]b Wa a i b]mgYhjb[gžcf Wcbgi a Yf dfUWljW]b Yj YfmXUm`]ZYžmci k]```YUfb`\ck hc UbU`mgY th Y g][b]Z]WUbW cZUWljcb UbX \ck hc XYg][b Yl dYf]YbWg th UhYb[U[Y Ui X]YbW g hc XUm" H\Y @]ZY cZDYfZcfa UbW []j Yg mci 'h Y g_]`g hc i bXYfgHUbX dYfZcfa UbW []b h Y WcbhYa dcfUfmk cf X" #iUddfcUW Yg h Y gh XmcZ dYfZcfa UbW Uwfcgg U VfcUX gdYWfi a 'Zfca 'gcW]U`f]hi U`g]b `cwU`gYhjb[g hc a i `hja YX]U Yj Ybhg cb th Y k cf X ghU[Y" H\fci [\ 'U Wca V]bUhjcb cZWUgY ghi X]Yg UbX dfUWljWU`Z]Y`Xk cf_žmci k]``Yl d`cfY \ck dUff]W]dUhjb[]b dYfZcfa UbWW WYYUhYg j U`i Ygž]XYbhjh]Yg UbX VY`]YZg h UhWcbbYWidYcd`Y hc[Yh\Yf UWfcgg [`cVU`bYhk cf_g"

7 ci fgY @YUfb]b['Ci hWca Yg

%" XYa cbglfUhY`Ub`i bXYfghUbX]b['cZh\Y`k UmdYfZcfa UbWY`ghi X]Yg`fY`UhYg`hc`ch\Yf`X]gW]d`]bYg`— YgdYW]U``mh\YUhfY`ghi X]Ygž'Ubh\fcdc`c[mUbX`W``hi fU``ghi X]Ygž'UbX`X]gh]b[i]g\`h\Y`]XYU`cZ 'dYfZcfa UbWY`Zfca 'h\Y`]XYU`cZ'h\YUhfY'

&" UbU`mgY`U`fUb[Y`cZYjYfmXUm`]ZY`VY\Uj]cifg`UbX`YjYbhg`Ug`'dYfZcfa UbW''

' " Udd`migca Y`cZh\Y`_YmihYfa]bc`c[mcZdYfZcfa UbWY`ghi X]Yg`—`gi W\`Ug``]a]bU`]miž`\m\f]X]miž 'ei ch]X]Ub'`UbX`'gma Vc`]WUbX`fYghcfYX`VY\Uj]ci f'—'hc`U`k]XY`fUb[Y`cZYj YfmXUmUbX WcbhYa dcfUfmidYfZcfa UbWY`dfUWf]WYgžUbX`VY`UV`Y`hc`Yj U`i UhY`h\Y`Yj c`j]b[`WcbbYWf]cbg`VYhk YYb h\cgY`dfUWf]WYg

("

YI dfYgg]cb" "Mci 'k]`` WfYUHY'dYfZcfa UbWY WcbWYdhg'h\UhfYZYWhi dcb'UbX'WY`YVfUHY'h\Y'k Umg']b'k\]W\dYfZcfa UbWY'ZYUhi fYg']b'ci f'XU]`m`]j Yg"

5ggYgga Ybh

 $\label{eq:decomposition} D`YUgY`gYY`h\ Y`WciifgY`A\ ccX`Y`Zcf`XYhU]`YX`]bZcfa\ Uh]cb`cb`WciifgY`UggYgga\ Ybhg"$

5ggYgga Ybh HUg_g

5ggYgga YbhHJg_	K Y][\h	8i Y [.] 8UhY	Ghi XYbhi@YUfb]b[(O how the hough of hour construction) (O how the hough of hour construction) (O how the hour construction)	s esse

5ggYgga Ybh' . \DX]j]Xi U`fYdcfh

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. *\\$\$\$ k cfXg

8YHJ]`g.

±x]j]Xi U`fYdcfhcb dfYgYbhUhcb"

%\$\$\$ k cfXg"

FiVf]WzWcaaYbhUbX^[fUXY"

5XX]h]cbU`XYhU]`g.

D`YUgY`gYY`UXX]h]cbU``]bZcfa Uh]cb`cb`h\Y`Wci fgY`A ccX`Y"

Gi Va]gg]cb bch/g. 18 i Y cb h\ Y Y j Yb]b['cZmci f'D]h\\ dfYgYbhUh]cb XUmVm \%da '! 'k YY_g' +! \%"

5ggYgga Ybh (. ⇒b!WUgg hYgh

8 YHJ]`g.

(\$'a]b"

; fUXY cb m

I	I	

D`YUgY`gYY`<u>I_BGK_'7cXY`cZ7cbXi_W</u>hZcf'a_cfY`]bZcfa_Uh]cb"'

FYWca a YbXYX FYgci fWfg

: fca 'f\Y'C b`]bY'F YUX]b['@]ghimci '\Uj Y'UWWgg'hc'h\Y'Wca d'YhY'Vcc_'; 'fcaVgUf[a` fa BWbAd_S`UW E fgV[We'VmF]W\UfX'GW\YW\bYf"'H\]g']g'U'\Y'd\Z\`'hYI Wcc_'hc'[fci bX'mci ']b'f\Y'X]gW]d']bY"

7 ci fgY '9j U'i Uh]cb UbX 8Yj Y'cda Ybh

Gh XYbhgʻ]bʻdfYj]ci gʻmYUfgʻYbʻcmYX``YUfb]b['UVci hdYfZcfa UbWYʻZfca 'X]ZZYfYbhdYfgdYWfj Yg"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb['tc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUh]cb'''#mci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k]''

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ "UbX GUZYmdc`]W]Yg/
- YI Ua]bUh]cb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX X]gUV]]hm/
- UbX'ch\Yf'YggYbh]U`'UWUXYa]W]bZcfa Uh]cbž'gYY

\hdg.#kkk"Ufng"i bgk"YXi "Ui #W ffYbhghi XYbhg#UWUXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

.