

ARTS3055

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 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong ``m5W_bck``YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJa di gŁż'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7Ua di gYgŁ'UbX'thY'B[i bbUk U` dYcd'Y'fbi gtfU']Ub'8YZYbWY': cfWY'5WJXYa m']b'7UbVYffUŁ'cZthY'`UbXg'k \YfY'YUW\cong WJa di g'cZl BGK ']g `cWJtYX''$

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Dcgg]V]`]hmhc`]bhYfj]Yk 'U``]hYfUfmdfUWh]h]cbYf

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
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a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9| hy'fbu''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Yl hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9| hy'fbu''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYf'Z'Vi h]Zmci 'hy'Yd\cby'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cby''
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; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

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- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
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Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

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