

ARTS3013

HYfa 'Hk c''##'&\$%

7ci fgY^{*}Cj Yfj]Yk

GHUZZ:7cbHUWh8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
9f]b [·] 6fUbb][Ub	· ·	Hi YgXUmg	F "'‰ K YVghYf 6i]`X]b[Ž) +,) (

GWXcc`7cbHJWh ±bZcfa Uh]cb

Fcca " ‰ž"Yj Y"" FcVYfhiK YVghYf'6i]`X]b[

D\cbY.'f\$&Ł'-',)'(,)*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. <u>\hdg.##gUa "Ufhg"i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd_S`UWEfgV[We

H\]g`Wci fgY`]g`cZZYfYX`]b`h\Y`H\YUlfY`UbX`DYfZcfa UbW'Ghi X]Yg`glfYUa "`±iWUb`U`gc`VY`ghi X]YX`]b`h\Y Zc``ck]b['gdYW]U`]gUh]cbg.`A i g]W

H\]g`Wci fgY`dfcj]XYg`mci `k]h\`Ub`cddcfhi b]hmhc`[U]b`]bXi ghfmYl dYf]YbW'`Vmi bXYfhU_]b[`k cf_!]bhY[fUhYX `YUfb]b[`cb`d`UWa Ybhk]h\`U'dYfZcfa]b[`Ufhg`cf[Ub]gUh]cb"`Ghi XYbhg`]bhYfYghYX`]b`[U]b]b[`k cf_ Yl dYf]YbW'`UfY`d`UWYX`k]h\`Ub`Uddfcdf]UhY`cf[Ub]gUh]cb`gi W\`Ug`GmXbYm8 UbW'`7 ca dUbmž`GmXbYm H\YUhfY`7 ca dUbmž`6Ub[UffUž: cfW'`A U'Yi fYž`7 f]h]WU``DUh\ž`DYfZcfa UbW'`GdUW'ž'I fVUb`H\YUhfY`Dfc'YWgž D57 Hž`DYfZcfa]b[`@]bYgž`6Y`j c]fž`D`Umk f]h]b[`5i ghfU`]Už`7 Uff]U[Yk cf_g`UbX`h\Y`I BGK `7 fYUh]j Y`DfUW]W' @UV`f7 D@L''

I dcb'Ybfc`a Ybhžmci 'a i ghgi Va]h'U'W ff]W `i a 'j]h'UY']bWi X]b['g_]``g'UbX'YI dYf]YbW'ž'UbX'U'ghUhYa YbhcZ dYfgcbU```YUfb]b['UbX'WUfYYf'[cU`g"'Mci 'k]``k cf_ 'cb'd`UW'a Ybhk]h\ 'Ub']bXi ghfmcf[Ub]gUh]cb'Zcf'-\$!%\$\$\ci fgža UbU[YX'Ufci bX'h\Y'cf[Ub]gUh]cbfg'fYei]fYa Ybhg'UbX'h\Y'ch\Yf'Wci fgY'fYei]fYa Ybhg'CZmci fXY[fYY"

 $\label{eq:continuous} $$ Mci bYYX`U`a]b]a ia `K 5A `cZ+$`]b`H\YUlfY`UbX`DYfZcfa UbW'`Gh X]Yg`cf`A ig]WglfYUa g`hc`VY`Y`][]V`Y`Zcf h\]g`Wci fgY"$

7ci fgY @YUfb]b['Ci hWca Yg

- %" F YWc[b]nY'h\Y'j Uf]ci g'ZJWYhg'cZh\Y'dYfZcfa]b['Ufhg']bXi ghfm'UbX'\ck 'hYUW\]b['UbX'`YUfb]b[ž WfYUh]j Y'XYj Y`cda Ybhž'dfcXi Whcb'UbX'UXa]b]ghfUhcb'UfY']bhYf!WcbbYWYX"
- &" 5dd`miU'j Uf]YhmicZWf]r]WU```YbgYg`hc'h\Y'fc`Y'cZd\mg]WU``dYfZcfa UbWY'Ug'U'hmdY'cZ_bck`YX[Y']b'h\Y VfcUXYf'UYgh\Yh]WZW'`hi fU`Z'gcW]U``UbX'YWcbca]WWcbhYl hg'h\Uha U_Y'i d'h\Y'dYfZcfa]b[''Ufhg]bXi ghfm'
- ' "8 Ya cbghfuhY fYgYufWX 'g_]``g`UbX`Wf]h]WU``h\]b_]b[']b`[Uh\Yf]b[ž`WUgg]Zm]b['UbX`Wca a i b]WUh]b[]bZcfa Uh]cb`UVci h'h\Y'dYfZcfa]b['Ufng"
- ("5dd`mik f]h]b['g_]``g'hc'h\Y'hfUbg`Uh]cb'UbX'Wca a i b]WUh]cb'cZdYfZcfa UbWY'dfUWf]WYg'UbX_bck `YX[Yg"
-) "8 Ya cbglfUhY'd\mg]\W\``Uk UfYbYgg`UbX`_bck `YX[Y'h\fci [\'dUfh]\W\dUh]cb']bz`cf`cVgYfj Uh]cb'c\Z \WfYUh]j Y'dfc\W'gg`UbX'dYf\Zcfa Ub\W'"

HYUWX]b['GhfUhY[]Yg'

7 YbhfU`WcbWidhg`Zfca 'h\Y'a U'cf`ghfYUa g`fBUbWi`Ghi X]YgžH\YUhfY`UbX`DYfZcfa UbWi`Ghi X]Ygž
A i g]WŁk]``VY`WUff]YX`]bhc`h\]g`gi V'YWhk\YfY`ghi XYbhg`k]``VY`Z]bY!hi b]b['g_]``g`]b`]bXYdYbXYbh
fYgYUfW\`i h]`]g]b['_bck `YX[Yg`UVWfi YX"

! 'hYUWX]b['UbX``YUfb]b[ž'WfYUh]j Y`XYj Y`cda Ybhž'dfcXi Wf]cb'UbX'UXa]b]ghfUh]cb'! 'UfY']bhYf! WcbbYWhYX"

- K f]h]b['hUg_g'k]```ZUW]`]hUhY'h\Y'UddfcUW\']b'Vch\'A U'cfg'hc'hfUbg`Uh]b['d\gm]WU``_bck `YX[Y'UbX dfUW]WY']bhc'k f]hhYb'hYl hg"
- #bMfbg\]dg'k]``VY`Uddc]bMX`VUgYX`cb'U'ghi XYbhfig'k cf_'UbX`]bMfYghg'Ug'h\Ym'\Uj Y`Ya Yf[YX UWfcgg'h\Y'%ghUbX`&bX'mYUfg'cZh\Y]f'A U^cf''
- H\Y`\ci fg`gdYbhik]h\`U`dUffbYf`cf[Ub]gUh]cb`k]``VY`ghfi Whi fYX`]b`Wcbgi `hUh]cb`k]h\`h\Y`Wci fgY Wcbj Ybcf"
- H\Y'7 f]h]WU``F YZYW]cbž'Dcfhzc`]c'UbX'7 J'K cf_g\cd'XfUk 'hc[Yh\Yf'k f]h]b[ž'dfUW]WY'UbX'Udd`]WUh]cb
]b']bXi ghfm'WcbhYl hgž'fYei]f]b['ghi XYbhg'hc'XYa cbghfUhY'd\mg]WU``Yl dYf]YbWY'UbX'`YUfb]b['Ug
 Ya VcX]YX'_bck `YX[Yž'Vi h'U`gc'hc'fYZYWh'UbX'Yl dUbX'cb'h\Uh_bck `YX[Y'h\fci [\'\]ghcf]WU``ž
 h\YcfYhWU``UbX'UbU`mh]WU\`k f]h]b["

5ggYgga Ybh

=k]```U]a 'hc'Wcbgi `hYUW\`cZmci ']bX]j]Xi U``mcb'YggUmhcd]Wg'UbX'dcfhZc`]c'WcbhYbhXi f]b['mci f']bhYfbg\]dž Vi hk]```gW\YXi `Y'U'[YbYfU``]bZcfa Uh]cb'gYgg]cb'Ufci bX'K YY__') "'=Zmci f']bhYfbg\]d'dYf]cX'YI WYYXg'h\Y dcfhZc`]c'XYUX`]bYžd`YUgY'VY']b'hci W\'k]h\'a Y"

K CF? GI 6A ≠H98.

5```Whuhcbgaigh]bWiXY'du[Y'fYZYfYbWrg"'Y['fbih\cfgifbuaY'MYUfz'du[Y'bc"ŁfGa]h\.&\$\$\$z')Ł

#D! HY I http://thombolographical.com/grufy.uww/dhuv.y"

: i ``V]V`]c[fUd\mUhh\Y`YbX`cZmciffYgYUfWX`dUdYfg`]g`fYei]fYX"

I bfY`]UV`Y k YVdU[Y fYZYfYbWg UfY bchUWWdHUV`YžY["K]_]dYX]U"`

: cbhg]nY'%&'d`YUgY'!'a m'YmYg'UfY'[Yhf]b['C@8"

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5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
+bHYfbg\]d'DcfhZc`]c'	')।	\$%\$,	%Ž Ž(Ž)
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7J'K cf_g\cd'	&) ı	% #\$*#&\$% `\$(.\$\$`DA	% <u>*</u> (

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5ggYgga Ybh% +bhYfbg\]d DcfhZc`]c

GHUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'b#J

8 YHU]`g. '5 'dYfgcbU``dcfhZc`]c`cZh\Y`]bhYfbg\]d'Zfca 'h\Y`gh XYbhg'dYfgdYWfj YžXcW a YbhJb['YI dYf]YbWYg UbX`dfc[fYgg'h\fci [\ci h'`H\]g`]g'h\Y`ZJbU``UggYgga YbhHUg_"A Uf_]b['U[U]bghU'[f]X`cZa Uf_]b['Wf]hYf]U`UbX k f]hhYb'Wca a Ybhg"`

5XX]h]cbU`XYhU]`g.

5 'dYfgcbU``dcfhzc`]c 'cZh\Y']bhYfbg\]d'Zfca 'h\Y'ghi XYbhfg'dYfgdYWf] YžXcW a YbhJb['Yl dYf]YbWYg'UbX dfc[fYgg'h\fci [\ci h''H\Y']bhYfbg\]d'dcfhzc`]c 'a][\h]bWi XY'fYdcfhg'k f]hhYb'cf'fYgYUfW\'i bXYffU_Yb'Xi f]b[mci f']bXi ghfmd`UWYa Ybhz'fYdfYgYbhUhJcbg'cZh\Y'k cf_d`UWY'gi W\'Ug'cf[Ub]nUhJcbU``UbX'Zck 'W\Uffg]XYbhJzmJb['UbX'XYgWf]V]b['ghUZZfc`Ygz'XYgWf]dhJcbg'cZYj Ybhg'UbX'mci f'fc`Y']b'h\Y'dfcWYggz'dfcdcgU`g'Zcf UWfj] JhJYg'Zcf'h\Y'cf[Ub]nUhJcbz'Zl bX!fU]g]b['d`Ubg'UbX'dfcYWhYX'ci hWca Yg'YhW''H\Y'dcfhzc`]c'g\ci 'X'fYWcfX mci f'Yb[U[Ya Ybhk]h\']bXi ghfmdYfgcbbY`z'YWh fYfgz'dYYfg'UbX'Ui X]YbWYgz'XYa cbghfUhJb['\ck 'mci '\Uj Y dUfh]WJdUhYX']b'UbY! W\Ub[Y'k]h\'h\cgY'mci 'UfY'k cf_]b['k]h\'"\Ci 'a UmU'gc']bWi XY'U'X]gW gg]cb'cZmci f

ck b`Yl dYf]YbWrg`k]th\`dYfZcfa UbWr`UbX`XUbWr`]b`fY`Ut]cb`tc`mci f`]bthYfbg\]d"`D`YUgY`]bWi XY`mci f`Z]bU``7 J]b`mci f`dcftYc`]c"

 $D`YUgY'dfchYWhmcif`\cghVmfYZYff]b[\ 'hc`U```dYfgcbbY``UbX'dfc`YWhg`Vm`YhhYf`fUh\Yf`h\Ub`bUaY''$

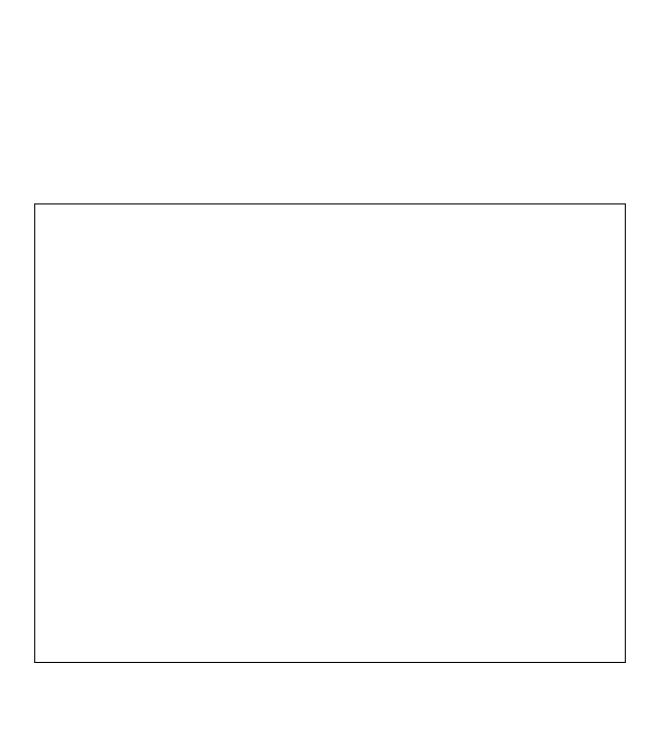
 $H\ [g'g\ ci\ X'VY'U'i\ gYZ\ ``fYgci\ fW'Zcf'Z\ 'i'\ fY'\ 'cV'Udd`]WUI]cbg'UbX']bXi\ gIfm'fYgYUfWX''$

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< 8	• 8 Ya cbglfUhYg'U'\][\`mgcd\]ghJWJHYX'i bXYfghJbX]b['cZh\Y cf[Ub]gUh]cb']b'ei Ygh]cb'UbX']rg'UW]j]hJYgž'Ub'UV]]lmihc i bXYfhJ_Y'gY`ZfYZYW]cb'UbX'UggYgga YbhUbX'h\Y']bWcfdcfUh]cb'cZZYYXVUW_Zfca 'a Ybhcfg'UbX'dYYfg"'9j]XYbWYg'UW]j YdUfhJWJdUh]cb'k]h\Y'\cghcf[Ub]gUh]cb'UbX'h\Y'i gY'cZUddfcdf]UhY'fYgYUfW\'fYgci fWYg"'J Yfmk Y``k f]hhYb'UbX fYZYfYbWYX"
8	• 8 Ya cbglfUhYg'U'gcd\]ghJWJhYX'i bXYfghJbX]b['cZh\Y cf[Ub]gUh]cb']b'ei Ygh]cb'UbX']hg'UWhj jhJYgž'Ub'UV]`]hmihc i bXYffu_Y'gY`ZlfYZYWh]cb'UbX'UggYgga YbhUbX'h\Y']bWcfdcfUh]cb cZZYYXVUW_Zfca 'a Ybhcfg'UbX'dYYfg"'9j]XYbWYg'dUfhJWJdUh]cb k]h\'h\Y'\cghcf[Ub]gUh]cb'UbX'h\Y'i gY'cZUddfcdf]UhY'fYgYUfW\X fYgci fWYg"'8Ya cbghfUhYg'[ccX'k f]h]b['UbX'fYZYfYbW]b['g_]``g"
7	• 8 Ya cbglfUhYg'U'[ccX'i bXYfglUbX]b['cZh\Y'cf[Ub]gUh]cb']b ei Ygh]cb'UbX']hg'UWhj]h]YgžUb'UV]']hmhc'i bXYfhU_Y'gY'ZlfYZYWh]cb UbX'UggYgga Ybh'UbX'h\Y']bWcfdcfUh]cb'cZZYYXVUW_'Zfca a Ybhcfg'UbX'dYYfg"'9j]XYbWfg'dUfh]WdUh]cb'k]h\'h\Y'\cgh cf[Ub]gUh]cb'UbX'h\Y'i gY'cZUddfcdf]UhY'fYgYUfW\'fYgci fWfg" 8 Ya cbglfUhYg'[ccX'k f]h]b['UbX'fYZYfYbW]b['g_]``g"
D	8Ya cbglfUhYgʻgca Yʻi bXYfghUbX]b[ˈcZh\Yʻcf[Ub]gUh]cbʻ]b ei Ygh]cbʻUbXʻ]hgʻUwh]j]h]YgžUbXʻgca YʻUV]`]hmhcʻi bXYfhU_YʻgYʻZl fYZYWh]cbʻUbXʻUggYgga YbhUbXʻh\Yʻ]bWcfdcfUh]cbʻcZZYYXVUW_ Zfca 'a YbhcfgʻUbX`dYYfg" 8Ya cbglfUhYgʻ[ccX`k f]h]b['UbX fYZYfYbW]b['g_]``g"
:	 8 Ya cbglfUllYgʻbcʻi bXYfglUbX]b[ˈcZlh\Yˈcf[Ub]gUlncbʻ]bʻei Yglncb UbXʻ]hgʻUWlj JhjYgž'UbXʻbcʻUV]`]mihcʻi bXYflU_YʻgY`ZlfYZYWljcbʻUbX UggYgga Ybh'UbXʻh\Yʻ]bWcfdcfUlncbʻcZZYYXVUW_Zfca 'a Ybhcfg UbX'dYYfg"

5ggYgga Ybh& =bXi ghfm9l dYf]YbW\#7f]h]WU`FYZYWh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&\$\$\$k



 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh"]g"gi Va]hhYX"h\fci [\ 'Hi fb]h]b"UbX"ghi XYbhg"Xc"bch"gYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

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GhUfhXUhY. Bch5dd`]WUV`Y

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

GYY`cb`]bY`fYUX]b[``]gh'Uh \hdg.##Jd\$%\U"U`a\U"Y\\`]Vf]g[fci\d"Wca\#Y[\Ubhc\#Y\UX]b[\`]gh#]ghg\#%*'\) +\$((,,\\$\$\$%+\%`

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GYY`cb`]bY`fYUX]b[``]gh'Uh' \hdq.#Ud\$%\U"U`a\U"YI\]Vf]q[fcid"Wca#Y[Ubhc#YUX]b[\]ah#]qhq#%*')+\$((,,\$\$\$%+'%]

7ci fgY '9j U'i Uh]cb 'UbX '8Yj Y'cda Ybh

 $H\ [g'WcifgY'k]'''VY'YjU'iUhYX'h\ fci[\ 'U'Am9IdYf]YbW''gifjYmUhh\Y'YbX'cZh\Y'WcifgY''$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybht\fci [\ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" Image:

UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUh]cb" Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k]''

Ui hca UhJWJ''mfYW'] Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYWJZWJ''mUg_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k]h\ci hUW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y a cghmfYhJ]b]b['h\Y'cf][]bU`gffi WhifY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa UhJcb'k]h\ci hUW_bck `YX[Ya Ybh''H\]g'U`gcUdd`]Yg']b'dfYgYbhUhJcbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]h'UbX'hc