

ARTS1092

K cf_]b['k]h\ '8 UhU

HYfa 'Hk c''##'&\$%-

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJh]cb	D\cbY
@][] 8] A Ufhjbc	`"X]a Uffi]bc4 i bgk "YXi "Ui			

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

8 UHU 'UbX']rg'f\Yrcf]WUfY'U```Ufci bX'i g"'H\Y'k Umg']b'k \]W\k Y']a U[]bY'XUHU'Ug'k Y```Ug'h\Y'XUHU'cV'YWrg h\Ya gY'j Yg'\Uj Y'U'dfcZci bX']a dUWrcb'h\Y'k Umg']b'k \]W\k t\Y'k cfX']g'cf[Ub]gYXž\ck fYgci fW'g'UfY X]ghf]Vi hYXž\ck k Y'i bXYfghUbX'fYWrbhYj Ybhg'UbX'k \Uhk Y'Wca Y'hc'_bck 'Ug'"ZUWr''j Yfgi g'a YfY "cd]b]cb""'±b'h\]g'Wci fgY'mci 'k]```YUfb'hc'Vch\'dfcXi WY'UbX'Wf]hJWU``mUbU`mgY'XUhU'cV'YWrg"'±b'UXX]h]cb'hc dfUWrJWJb['XUhU'Yl hfUWr]cbž'UbU`mg]g'UbX'j]gi U`]gUh]cbžmci 'k]````YUfb'\ck 'hc'Yj U'i UhY'h\Y'a i 'hJd'Y YI Ua d'Yg'cZXUhU'h\Uhmci 'YbWci bhYf'"]b'h\Y'k]x'"-XUhU'h\UhUfY'i gYX'Vmh\Y'a YX]UžVm[cj Yfba Ybhg'UbX WJj]`gcWJYmhc'Yl d'U]b'UbX'cf[Ub]gY'h\Y'k cfX'Ufci bX'mci "'6mih\Y'YbX'cZh\Y'Wci fgYžmci 'k]```\Uj Y'[U]bYX U'gc`]X']bhfcXi Wr]cb'hc'h\Y'dfcW'ggYg']bj c'j YX']b'k cf_]b['k]h\`XUhU'UbX'Ub'UV]`]hmhc'fYZYWricb'W ffYbh XYVUhYg'UVci hh\Y']a dUWrcZXUhU'cb'gcWJYm'

7 ci fgY @YUfb]b[Ci hWca Yg

%" 91 hfuWmubu`mgY`UbX`j]gi U`]gY`XUhU`]b`a YUb]b[Zi `žYh\]WU``k Umg &" 7 f]h]WU``mYj U`i UhY`XUhU`cV`YWng '' " F YZYWncb`W ffYbhXYVUhYg`UVci hh\Y`]a dUWncZXUhU`cb`qcW]Yhm

HYUWX]b['GhfUhY[]Yq'

5ggYgga Ybh

 $\label{thm:continuous} $$H\YfY\UfY\th c`UggYgga\YbhHUg_g'hc`Wca d`YhY''Mci 'a i ghWca d`YhY`UbX`\UbX`]b`Vch\`HUg_g']b`cfXYf'hc dUgg'h\]g'Wci fgY'''H\]g'a YUbg'h\Uhmci 'k]```ZU]`']Zmci 'Xc'bch\UbX']b`cbY'HUg_'Yj Yb']Zmci '\Uj Y'U'hcHU` [fUXY`cZcj Yf') $`Zfca 'h\Y`ch\Yf'Ugg][ba Ybh'$

GYY'h\Y'WcifgY'kYVg]hY'cb'AccX`Y'Zcf'Z```'Ugg][baYbhHUg_'XYgWf]dh]cbgžaUf_]b[''Wf]hYf]U'UbX'giVa]gg]cb]bghfiWf]cbg"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_		

 $\label{eq:continuity} D`YUgY`gYY`A\ ccX`Y`Zcf`Z\ ```UggYgga\ Ybh`XYgWf]dh]cb"$

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b"gYh]b"[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \end{tabular}$

5 hmy b X Ub WY F Yei] f Ya Yb hg

5 ````YWi fYg`]b'h\]g'Wci fgY'UfY'cb`]bY"

Gh X]c 'fh hcf]U' E' WUgg 'Ulth' b XUb W' ']g 'Wca di `gcfm`]b 'k YY_g', ž- 'Ub X' %\$ 'Ug 'mci 'k]```VY 'Wca d`Yh]b['UWf]j]h]Yg]b 'dfYd Uf Uh]c b 'Zcf'h Y 'Ugg Ygga Yb th' Ghi XYb hg 'Uf Y' Yl d YWf X' hc 'Ulth' b X' U```ch Yf'k YY_g' ffYl WYdh 'Zcf'h Y F YUX]b['K YY_']b 'k YY_'* E'' H \ YgY ']b! WUgg 'UWf]j]h]Yg 'k]```\Y`d 'mci 'hc 'Ulth']b 'U``'h fYY 'Wci fgY '`YUfb]b[ci h\can Yg'' l b Yl W gYX 'UVgYb W 'Zfca 'a cfY'h Ub', \$1 'cZGhi X]c 'WUgg Yg 'k]```fYgi `h']b 'h Y 'Uk UfX' U'ZU]` [fUXY''

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YHJV`Y

8UHY	HmdY	7 cbhYbh	
K YY_'% '' '>i bY '! '+' >i bY	@YWi fY	⇒hfcXi Whjcb'-'8 UHJ'UbX'j]gi U`]gUhjcbg	
	Gh X]c	7 \ccg]b['h\Y'a cgh'Uddfcdf]UhY'j]gi U`]gUh]cb	
I/			
K YY_*&.*%\$*>i bY*!*%(@YWifY	A Yh\cXg`Zcf`UbX`]a d`]WUh]cbg`cZ[Uh\Yf]b[`XUhU	
>i bY	Gh X]c	8 UHJ YI HTUWIjcb. Hk jHYf	
K YY_'' . '%+ '>i bY '! '&%	@YWhifY	H\Y`]a dUWicZXUhU`cb`gcWJYhm`XYWJg]cb`a U_]b['UbX	
>i bY		_bck`YX[Y`dfcXiWf]cb	
	Gh X]c	8 UHU YI HTUWHJcb.: UWYVcc_'dU[Yg'UbX'; cc[Y'BYk g	
K YY_'(.'&('>i bY'!'&,	@YWhifY	6]['8 UHJ '9 h\]\\g*\Df]j U\\n*\Z*UbX 'Gi fj Y]``Ub\\\Y'	
>i bY	Gh X]c	K_cf_]b['k]th '91 WY`'UbX'a Ubi U`'WcX]b['	
K_YY_^) . "%">i `m'! ") ">i `m	@YWhifY	?Yma Yhf]Wgʻi gYXʻZcfʻUbU`mg]b['Hk]hhYf	
	Gh X]c	K cf_]b['k]h\ 'HUV`YUi '	
	5ggYgga Ybh	9ggUmXi Y`h\]g`k YY_"	
K YY_'*.', '>i `m'! '‰'>i `m	FYUX]b[FYUX]b['K YY_'! 'Bc'@YWhifYž'Bc'GhiX]c'7`UggYg	
K YY_'+.'%) '>i `m'! '%- '>i `m	@YWhifY	J]giU`]g]b['BYhkcf_g'!`+bhfcXiWhjcb'hc'h\Y'Dcfh7c`]c'	
	Gh X]c	K cf_]b['k]h\'; Yd\]	
K YY_', .'&&'>i `m'!'&*'>i `m	@YWhifY	7 fYUhY mci fij]gi U`]gUh]cbi! i±bhfcXi Wh]cbihcih\Y	
		Dcfh7c`]c	
	Gh X]c	K.cf_'cb'mci f'dcfhZc`]c	
K YY_''&-'>i `m'!'&	@YWhifY	@YUfb]b['Zfca 'dfUWf)WYg'	
5i[igh	Gh X]c	K cf_'cb'mci f'dcfhZc`]c	
K YY_"%\$.") "5i [i ghi!"-	@YWhifY	Gia!id.~Kcf_]b[~k]h\~XUhU~]g~U~kUmg~U~~YUfb]b[
5i [igh		dfc W gg	
	Gh X]c	; Yth]b["fYUXmZcf"gi Va]th]b["mci f"DcfhZc"]c	
	5ggYgga Ybh	Dcfh7c`]c'XiY'h\]g'kYY_"	

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

5```k YY_`mfYUX]b[g`UbX`ch\Yf`Wci fgY`WcbhYbhk]```VY`Uj U]`UV`Y`j]U'h\Y`Wci fgY`k YVg]hY`cb`A ccX`Y"

FYWca a YbXYX FYgci fW/g

H\Y`I BGK @VfUfm\Ug`a Ubmi gYZ ``fYgci fWrg`cb`XUhU`UbX`j]gi U`]gUh]cbgž'Zcf`YI Ua d`Y.

5 W. UfmUž'G"/ '7 \Y``UddUbž'G"ž'&\$%+""Bda FST WSg, 3 EfWbŽTkŽEfWb 9 g[VWž'6Yf_Y`Ymž'75. '5dfYgg"

6i f[Yggž'>"ž'A Ufk]\"\Z'5""9"\"\Z'\DcY\\\Z'H"\f\&\\$\\+\L"FZWE 39 7 ZS\\VTaa] aXeaU[\S_\W[\S\\\@cg'5b[\Y\\Yg. \G5; 9"

; Ultrož'A "'5" f&\$%) Ł"'? S] [Y dWeWSdUZ geWg, 5 gddW f UZS^W YWe S` V YaaV bdSUf[UWe [VSfS h[egS^[eSf[a` " C | ZcfX. F Yi hYfg' \pm 0 gh]hi hY 'XadfZWE fgVk aX<agd S^[e_ ž

 $B[i\ mYbz'5"z'\&\$\%""@W'\ el`g_TWde\ S`V\ bgT\{U\ ab[[a`[SVSfSZVd[hW]i\ adVz'BYk'Mcf_z'BM'6`cca\ gVi\ fm'5WUXYa]W'$

G`cUbž@'ž&\$%+"`FZWE397ZS`VTaa]aXeaU[S^_WV[SdNeWSdJZ_WfZaVež@cbXcb.`G5;9"

 $K_{Y}^{T}fz^{2} = 26 \text{ for } 16 \text{ for$

7 ci fgY '9j U'i Uhcb 'UbX '8 Yj Y'cda Ybh

H\]g`Wci fgY`]g`XY`]j YfYX`Zcf`h\Y`Z]fghh]a Y`]b`&\$% "`D`YUgY`Wca d`YhY`mci f`a m9I dYf]YbWY`gi fj YmiUhh\Y`YbX cZhYfa 'hc`\Y`d`i g`XYj Y`cd`h\Y`Wci fgY`Zcf`Z hi fY`gh XYbhg"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY