

# **MUSC2115**

A i g]WcZhY'% hY'7Ybhifm

HYfa 'C bY''##'&\$&%

## 7ci fgY'Cj Yfj ]Yk

### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
8cfcHmU: UV]Ub	X"ZUV]Ub4 i bgk "YXi "Ui	VmWcbgi `HJh]cb	K YVghYf"%\$'	-',)*-)(

### GWXcc~7cbHJWh~abZcfa Uhlcb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY. f\$&Ł'-',) '(,) \*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$ 

## 7ci fgY 8YW] g

7 fYX]hDc]bhg\*\*

### Giaa UfmcZh\Y 7ci fgY

5 'ghi XmcZ% h\!W'bhi fm'9i fcdYUb'a i g]W]b']hg'\]ghcf]WU'z'gcWJU''UbX'W 'hi fU''WcbhYl hg"'\bW XYg'U'hi hcf]U'']b a i g]Wc'c[]WU''fYgYUfW.'hYW.b]ei Yg'UbX'X]gW gg]cb'cZ\]ghcf]WU''dYfZcfa ]b[ 'Wcbj Ybh]cbg"

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 8 Ya cbghfuhY ZJJa ]`]Uf]hmik ]h\ 'h\ Y`a cghfYdfYgYbhJh]j Y`Wca dcg]h]cbg`cZh\ Y`F ca Ubh]WdYf]cXž ]bWi X]b[ `cdYfUž@]YXYfž'gc`c`UbX`W.Ua VYf`a i g]Wž'UbX`cfWXYghfU``k cf\_g"
- &" 8 ]gW gg'h\Y'\]ghcf]WJ`g][ b]Z]WUbW'UbX'ghm]gh]WZYUh fYg'cZ% h\ 'WYbh fm'Wca dcg]h]cbg']b'UbUfh\W `UhY'UbX']b\Zcfa YX'a UbbYf"
- ' "8]gd`UmUWei]fYX`\_bck`YX[Y`UVci ha i g]WUbX`gcW]Yhm]b`% h\!WYbhi fm9i fcdY`UbX`h\Y`fc`Y`% h\! WYbhi fmUYgh\YhJWgž]bhY``YWhi U``]XYUg`UbX`k cf`X!j]Yk 'd`UmYX`]b`g\Ud]b[ 'a cXYfb`a i g]Wb`c[]WU`

### 5ggYgga Ybh

### 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'hUg_	K Y][ \h	8i Y <sup>.</sup> 8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
9ggUmi	(\$1	&&#\$(#&\$&%`\$).\$\$`DA</td><td>&ž'</td></tr><tr><td>' ``]ghYb]b[ 'hYghg'</td><td>* \$1</td><td>Bch5dd`]WUV`Y</td><td>%ž (</td></tr></tbody></table>	

### 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% '9ggUm

GHJfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&\$\$\$'k cfXg

8 YW] g.

&\$\$\$ k cfXgžZYYXVUW\_dfci ]XYX ]b h fb]hb"

#### 5XX]h]cbU`XYhJ]`g.

9ggUmg`\Uj Y`hc`\Uj Y`Ub` ⊕hfcXi Whjcbž8]gW gg]cb`UbX`7 cbWi g]cb"'H\YmbYYX`hc`Wf]hJWU``mYb[ U[ Y`k ]h\ gWkc`Uf`mgci fWyg`UbX`fYZYfYbWY'h\Ya 'UVWcfX]b[ 'hc'h\Y`5D5`ghmY"'H\Y`X]gW gg]cb`g\ci `X`XYa cbghfUhY ZJa ]`]Uf]hmk]h\`a i g]WJ``Yl Ua d`Yg`cZmci f`W\c]WY'!!`]"Y"`gY`YWhmci f`ck b`a i g]WJ``Yl Ua d`Yg`hc`Yl Ya d`]Zm k\Uhmci `\Uj Y``YUfbhZca 'UbX'UfY 'X]gW gg]b[ 'cZh\Y`gWkc`Uf`m`]hYfUhi fY"

: i fh\Yf`]bZcfa Uh]cb`UbX`U`gY`YWh]cb`cZgi [[YghYX`hcd]Wg`k]``VY`dfcj]XYX`Uh'h\Y`ghUfhcZhYfa`]b`A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh]g"gi Va ]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

5ggYgga Ybh& " ``]ghYb]b[ 'hYghg

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`q.

@]ghYb]b[ 'hYghg'!'g\cfhYl WYfdhg'#Ubgk Yfg'!'a i g]WfYWc[b]h]cb'UbX'\_bck `YX[Y'fl \$'a ]bi hYgŁžk f]hhYb'/j YfVU``ZYYXVUW\_"

#### 5XX]h]cbU`XYhU]`g.

 $G\backslash cfh'Ui X]c'YI Ua d'Yg'k ]^*VY'd'UmYX'gY'YWYX'Zfca 'h\Y'`]ghcZk cf_g'Ugg][ bYX'Zcfa i g]W']ghYb]b[ "'Mci 'k ]^*VY'Ug_YX'hc']XYbhJZm'h\Y'Wca dcgYfz'h]hYz'[ YbfY'UbX'UddfcI ]a UhY'XUhY'cZWca dcg]h]cb'''Mci 'k ]^*'U'gc'VY fYei ]fYX'hc'k f]hY'cbY'cf'kk c'gYbhYbW'g'YI d'U]b]b[ 'h\Y'a i g]WU''cf'\]ghcf]\WU''g][ b]ZJ\WUW''cZh\Y$ 

Wca dcg]h]cb"

 $\hbox{Gi Va ]} gg] cb \ bc \ h \ g. \ \ ] b! \ W \ Ugg \ h \ ghg \ cb \ k \ Y \ Y \ g. \ (\ \check{z}' + \ Ub \ X' \% )$ 

Hi fb]h]b gYh]b[. H\]g ]g bchU Hi fb]h]b Ugg][ ba Ybh

## 5 hm/b XUb VW F Yei ]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$ 

# 7ci fgY GWX YXi `Y

## J]Yk 'WUgg'hja YhUV'Y

## H]a YtUV`Y

8 UhY	Hmd Y	7 cbhYbh
K YY_'%'%)': YVfi Ufm!'%- : YVfi Ufm		@YWnffY.'<]ghcf]WU``VUW_[fcibX'hc'aig]W]b'h\Y 7%h\
		FcaUbhjWjgaʻ]bʻaig]WʻAig]WJ``[YbfYgʻUbXʻghm`]ghjW VX.UfUWhYf]ghjWg
		Hi hcf]U`g. 6YYh\cj Yb'g'' fX`UbX`*h\`Gma d\cb]Yg/ K YVYf. 8Yf': fY]gW\ØmžK c`Z; `Yb`GW\bY
K YY_'&.'&&': YVfi Ufm!'&* : YVfi Ufm		@YWnffy."CfWkYghfUg"UbX"cfWkYghfUh]cbz"hkY "aYUb]b[""cZ]bghfiaYbhU`"Wc`cifg
		Hi hcf]U`g. 'Gma d\cbm': UbhUgh]ei Y'fbYf']cnlž'H]`` 9i 'Ybgd]Y[Y`'fF]W\UfX'GhfUi gglž'6fU\a gž'A U\'Yf UbX'K U[bYf
K YY_" ."%"A UfVX"!") A UfVX		@YWnffy.`>[WWžU'ei]bhYggYbh]U`FcaUbh]WUfhZcfa
		Hi hcf]U`g. 'H\Y'Gcb[ g'cZGW\i VYfhž'GW\i a Ubbž 6fU\a g'cf'K c`ZUbX'F "'GhfUi gg
K YY_'(.', 'A UfVX'!'‱ A UfVX		@]ghYb]b[ 'hYgh'%'!']b'Hi hcf]U`'fCfW.YghfU`'UbX'@]YXL
		@YWnifY.`D]Ubc`aig]W]b`h\Y`%h\`WYbhifm`="
		Hi hcf]U`g.`A ]b]Uhi fY`d]Ubc`d]YWYg/DYfZcfa UbWY dfUWf]VW
K YY_^).^%)^A UfWX^!^% A UfWX		@YWnfY.'D]Ubc'aig]W]b'h\Y'%h\'Whffm'="

	Hi hcf]U`g.`⊭U`]Ub C dYfUž'J YfX]
K YY_', .') '5df]`!'- '5df]`	@YWhifY. KU[bYf"
	Hi hcf]U`g.`K U[ bYf
K YY_''\%\'5df]`'!'\%*	@YWhifY."CdYfU"]b"h\Y"gYWcbX"\U"ZcZh\Y"% h\
5df]`	Wbh fm
	Hi hcf]U`g. J YfX]ž'6]nYhž'Di WWJb]'fK U[ bYfŁ
K YY_'%\$.'%-'5df]`'!'&'	@]ghYb]b[ 'hYgh' '!'ftdYfUt"'
5df]`	
	@YWhifY."FYj]g]cb"UbX"GiaaUfm
	Hi hcf]U`g. FYj ]g]cb'UbX``]ghYb]b[ 'hYgh

### FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

5XX]h]cbU`i gYZi ``"gi fj Ym" Vcc\_g.

@cb[mYUfžFUmžA.B]bYhYYbh\!WYbhfmfcaUbh]W]ga]baig]W'9b[YkccX7]ZZgžB">"žDfYbh]WY!<U``ž%\*-

D`Ubh]b[ Už@Ycb. F ca Ubh]Wa i g]W! 'U`\]ghcfmcZa i g]WU``ghm`Y`]b`b]bYhYYbh\!WYbh fmi9i fcdY"'BYk 'Mcf\_'. K "K "Bcfhcbž'%, (

F UrbYfž@YcbUfX. `F ca Ubr]Wa i g]W! `gci bX`UbX`gmbrUl "`BYk `Mcf\_`. `GW\]fa Yf`6cc\_gž`% - &"

: i ft\ Yf fYgci fWyg k ]``VY dfcj ]XYX ]b WUgg UbX A ccX`Y"

### FYWca a YbXYX FYgci fWyg

 $GWcfYg'UbX'fYWcfX]b[g'cZkcf\_g'gh'X]YX''$ 

5b'YggYbhJU```]gh']bWi XYg'h\Y'Zc``ck ]b[']hYa g.

C fWX YghfU`.

6YYh\cjYb. Gma d\cbmBc"' f9fc]WUŁ

6Yf]cn. Gma d\cbm: UbHJgh]ei Y

6fU\a g. Gma d\cbmBc" % (h\ a cj Ya Ybh

A U\`Yf. `Gma d\cbmBc"') ž(h\`a cj Ya YbhfbXU[]YhcŁ

F ]W\UfX'G\fUi gg. 'H]\`'9i \Ybgd]Y[Y\

Gc`c'D]Ubc.

@gnh 6 a lbcf GcbUHJ

 $7 \ Cd]b. \ BcW \ fbY \ Cd" \ - \ Bc" \ \& \ / \ GU``UXY \ Bc" \ \%]b'; \ \ 'a \ ]bcf$ 

GWXi a Ubb. 7 UfbUj U`

@YXYf.

GWXi VYfh 'H\Y'9f\_]b[ 'f9f\_cb][ \text{\form} 'gY\YWYX'gcb[ g'\text{\form} Tca 'K ]b\YffY]gY'

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

# 5WUXYa]W±bZcfaUhjcb

: cf`YggYbh]U``gh`XYbh]bZcfa~Uh]cb`fY`Uh]b[~hc.

- fYei Yghg Zcf Yl hYbg]cb/`UhY gi Va ]gg]cbg [ i ]XY`]bYg/