

MDIA5006

: YUhi fY'K f]h]b[

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y'7ci fgY

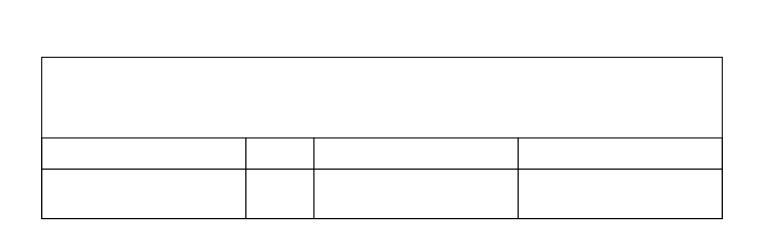
H\]g`Wci fgY`Vi]`Xg`cb`f\Y`WcfY`K f]hjb[:: cf`A YX]U`Wci fgY`cZYfYX`]b`HYfa '%z`k\]W\`HYUW\Yg`f\Y\
Z bXUa YbhU`g`cZk f]hjb[`Zcf`df]bh'ci fbU`]ga "'=b`f\]g`Wci fgYžmci `k]```YUfb`\ck 'hc`k f]hY`ZYUhi fY`Uff]WYg"
Mci `k]```XYj Y`cd`mci f`]bhYfj]Yk ž`fYgYUfW\`UbX`k f]hjb[`g_]``g`Zcf`Vch\`df]bhUbX`cb`]bY`a YX]U"`Mci `k]```YUfb
hYW\b]ei Yg`Zcf`k f]hjb[`]b`U`fUb[Y`cZZYUhi fY`Uff]WY\[YbfYg`UbX`Zcfa g"'H\]g`Wci fgY`k]```U`gc`Wcj Yf`h\Y`fc`Yg
cZYX]hcfg`UbX`ZfYY`UbWf`k f]hYfg`]b`dfcXi Wjb[`ZYUhi fY`Uff]WYg"`Mci `k]```ghi Xmh\Y`\]ghcfmUbX`h\Y
WcbhYa dcfUfmgcWjU``g][b]Zj\Wb\Wf`cZh\Y`ZYUhi fY`[YbfYg"`Mci `k]```Wf]hj\WJ``m\Wcbg]XYf`_Ymk f]hYfgžhYI hg`UbX
Ui X]Yb\Wfq`Zcf`h\YgY`[YbfYg"`

7 ci fgY @YUfb]b[Ci hWca Yg

- %"; YbYfUhY`ZYUhi fYg`cZj Ufm]b[``Yb[h\g`cb`XYUX`]bYžhc``Yb[h\ž'WcbZcfa]b[`hc`fYei]fYa Ybhg`cZ X]ZZYfYbh[YbfYg`]b'j Uf]ci g`a YX]U'ci hYhg
- &" D`UbžifyqYUfW\žighfi Whi fy'UbXik f]hy'ZYUhi fyqžiei Yf]Yq'UbXid]hW\Yq
- "FY`mcb'kY``!XYjY`cdYX`fYdcfh]b['UbX'bYkg'[Uh\Yf]b['g_]``g
- ("I bXYfghJbX'dfcZYgg]cbU``YX]hcf]U``dfcW/ggYgžYh\]Wg`UbX'h\Y'XYa UbXg'cZh\Y'a Uf_Yhd`UW/
-) "Kcf_'Ug'dUfficZU'HYUa 'hc'WfYUHY'ZYUhi fYg'Zcf'a U]bglfYUa 'a YX]U
- *" HU_Y`cb`h\Y`W\U``Yb[Y`cZbYk`ghcfmihY``]b[`'hYV\\b]ei Yq

HYUWX]b[GhfUhY[]Yg

 $H\setminus]g`Wci fgY`Wca V]bYg`C B 9`hk c!\ci f`gYa]bUf\#k YY_`K ++< `ga U```UbX``Uf[Y`[fci d`X]gWi gg]cbgž`]b!WUgg k cf_g\cdgžYj U`i Uh]cbžYl YfW]gYg`VUgYX`cb`Wci fgY`fYUX]b[gž`\UbXci hg`UbX`gYa]bUf`WcbhYbhž'UbX`Zci f UggYggYX`hUg_g`k]h\`XYUX`]bYg"``$



5XX]h]cbU`'XYhU]`g.

H\Y', \$\$!k cfX`ZYUhi fY`f\) \$!,) \$`k cfX``]a]h\a i gh\Y'k f]hh\b`]b`cbY`cZh\Y`Zc``ck]b[`[YbfYg.`F Yj]Yk ž Cd]b]cbž5bU`mg]gž9l d`U]bYf`cf`\a a Yfg]cb"\Mci f`gi Va]gg]cb`a Um\VY`]b`h\Y`Zcfa `cZh\I h\u00ecUi X]c fdcXWJgh\u00eccfj [XYc`f] cXWJgh\u00eccfj [XYc`f] cXWJgh\u00eccfi = a Yfg]cb"\Mci f`gi Va]gg]cb`a Um\u00eccfi | X\u00eccfi | X\

Gi Va]gg]cb bchYg. GYY [YbYfU`UggYgga Ybh]b/cfa Uh]cb

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh"]g"gi Va]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{array}$

5ggYgga Ybh& @cb[: YUh fY

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'%(\$\$!%*\$\$'k cfXg

8 YW] g.

%) \$\$ k cfXgžei U`]hUhj Y ZYYXVUW_dfci]XYX i]U Hi fb]hjb"

5XX]h]cbU`'XYhU]`g.

H\Y'@cb[': YUhi fY'a i gh'VY'k f]hh'b']b'cbY'cZh\Y'Zc``ck]b['[YbfYg. BYk g'cf'=ggi Y!VUgYX'ZYUhi fYžDfcZ]`Yž@]ZYghmY#HfYbX'fYdcfh'cf'=a a Yfg]cb'''Nci f'gi Va]gg]cb'a UmVY']b'h\Y'Zcfa 'cZhYl hž'Ui X]c'fdcXWUghz'cf'j]XYcf] cXWUghz

Gi Va]gg]cb bchYg. GYY; YbYfU`5ggYgga Ybh \$\mathbb{Z}cfa Uh]cb

Hi fb]h]b gYh]b[. `H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a]`Uf]hmfYdcfhg"

5ggYgga Ybh' . : YUhi fY'K f]h]b['UbX'F YgYUfVX 'DcfhZc`]c

@Yb[h\. %(\$\$!%*\$\$kcfXg

8 YHJ] g.

%) \$\$ k cfXgžei U`]hUhjj Y ZYYXVUW_dfcj]XYX j]U Hi fb]hjb"

5XX]h]cbU`XYhU]`q.

 $DcfhZc`]c`gi Va]gg]cbgZ'k \]WX`UWWci bhZcf'') `dYfWYbhcZh\Y`Uj U]`UV`Y`a Uf_g`]b`h\]g`Wci fgYZ`WUbbch]bWi XY a UhYf]U``gi Va]hYX`]b`5gg][ba Ybhg`C bY`UbX`Hk c"`Ghi XYbhdcfhZc`]cg`a i ghWcbhU]b`cb`mh\Y`hYI hZ`]a U[Yg$

 $GI 6A + GG + B "'H\Y' mdYg'cZa UHYf]U''h\Uha UmVY'gi Va]HYX']bWi XY'dfY!WUgg'UbX']b!WUgg'gi Va]gg]cbg cf'HYI HVUgYX'UW]j]HYgžghcfmd]hW\YgžfYZYW]cbg'cb''HUg_g'UbX'fYUX]b[gžfYZYW]cbg'cb']bHYf]]Yk ei YgH]cbg'cf'fYgYUfW\'gHfUHY[]Yg'h\UhdfcXi WYX'bYk gk cfh\ma UHYf]U'ž'UbX'ch\Yf'_]bXg'cZk cf_"'Ghi XYbhg UfY'fYgdcbg]V'Y'Zcf'W\ccg]b['UbX'gi Va]H]b['cb'mh\Y]f'VYghk cf_'Zcf'h\Y]f'dcfhZc']c"'Dccf'W\c]W'g']b'h\]g fY[UfX'UfY'bch'[fci bXg'Zcf'UddYU'"$

Gi Va]gg]cb bchYg. GYY; YbYfU`5ggYgga Ybh \$\dagger Zcfa Uh]cb

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh']g'gi Va]hh/X'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb g]a]`Uf]hmfYdcfhg"

5 https://bxubwww.Fyei]fya Ybhg

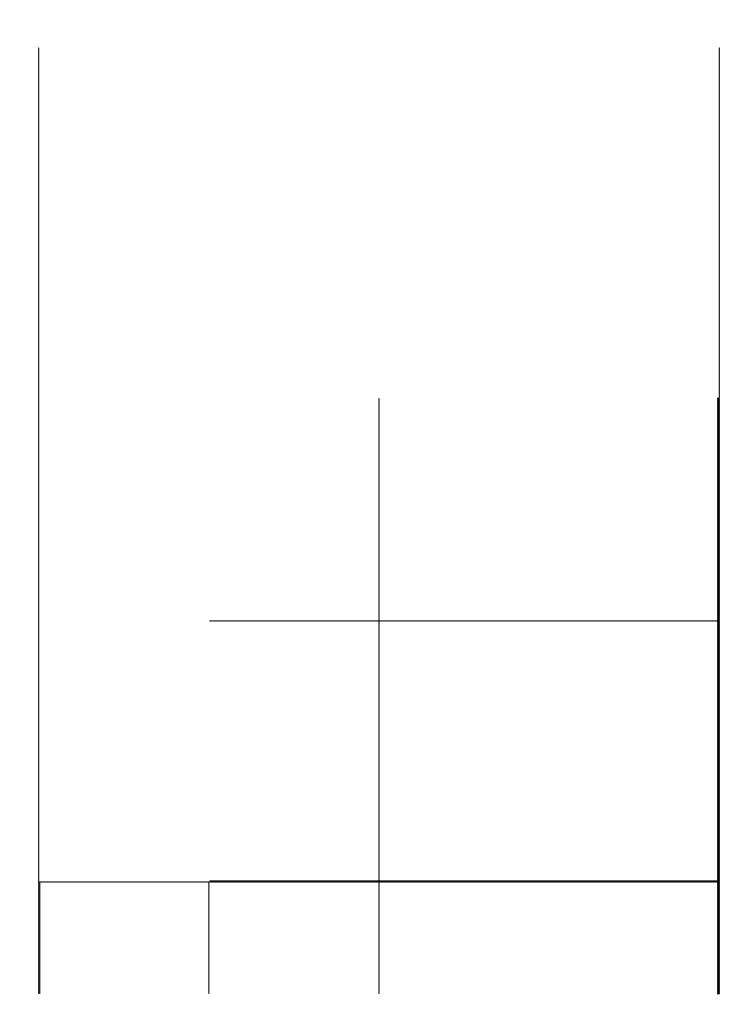
Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YhUV'Y

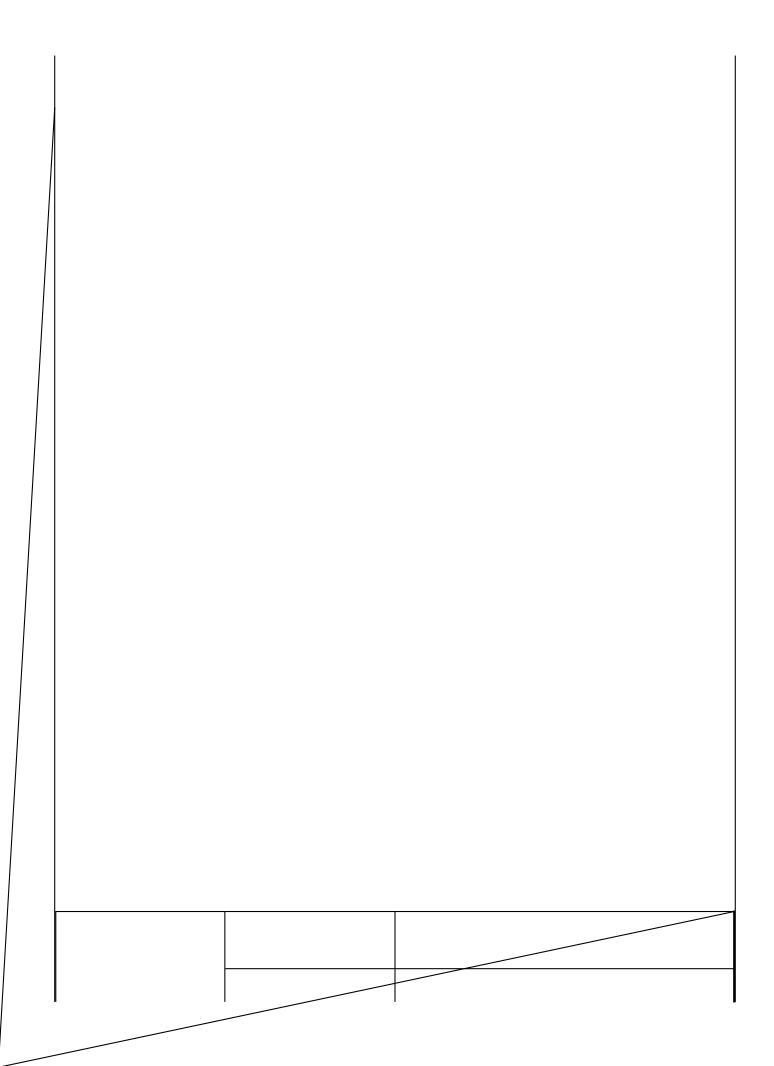
H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh
K YY_'%'%) ': YVfi Ufm'! '%- : YVfi Ufm	< ca Yk cf_	69: CF9'7CA B; 'HC'7@5GG. 'Gh XYbhg'g\ci `X UWW'gg'h\Y'; 9BF9'@6F5FMcb'h\Y'AccX'Y Wci fgY'g]hY'Zcf'Yl Ua d'Yg'cZZYUh fY'Ufh]WY'[YbfYg VYZcfY'W\ccg]b['h\Y'[YbfY'h\Ymk]`'UfhYa dh'Zcf 5gg][ba Ybh'CbY'fcd]b]cbz'UbU'mg]gz'Yl d'U]bYfz fYj]Yk'cf']a a Yfg]cbŁ'''D'YUgY'U'gc'j]Yk'h\Y Zc''ck]b['j]XYcg.
		K \Uh`]Zk Y`\Y`dYX`fYZ [YYg`hc`\Y`d h\Ya gY`j Yg3 ``5`Yl UbXYf`6Yhtgž% "%%"%) H981 J]YbbU` \htdg.#k k k "mci h VY"W:a #k UhW.3j 1 hK k fXA ej j A E i]h'GcW]U``A YX]U ž8f`7 U``BYk dcfhž&\$"-"%* ž
		H981 Hmgcbg` \hdg.#kkk"mci hi VY"Wca #k UhWk3j 1'9+\ DN! <h_< td=""></h_<>
	5ggYgga Ybh	K 99? '%DF 9!7@5GG'GI 6A €G€B fDCFH: C@€'%Z&\$\$'k cfXgŁ'6fck gY'h\Y'cd]b]cb Ufh]WYg']b'h\Y'; YbfY'@]VfUfmcb'A ccX'Y"'7\ccgY cbY'cZh\Y'HYX'HU'_g'UVcj Y'UbX'gi Va]h&\$\$'k cfXg cb'k\Uhh\Ym\Wb'hYUW\'i g'UVci hYZY\W]j Y UddfcUW\Yg'fc'h\Y'cd]b]cb'[YbfY'cZYj U'i Uh]j Y ZYUh fY'k f]h]b["'K\Uhrcd]Wa][\hmci 'W\ccgY']Zmci k YfY'fc'k f]hY'Ub'cd]b]cb'Ufh]WY3'
		8958@-B9. ``) `da `A CB85M%'> B9`&\$&\$ G 6A +HJ-5 `H<9`A CC8@9`DCFH: C@-€ `%@-B? B89F`H<9`5GG9GGA 9BH`H5;
K YY_'&.'&&': YVfi Ufm!'&* : YVfi Ufm	< ca Yk cf_	69: CF9'7CA B; 'HC'7@5GG. 'Ghi XYbhg'g\ci 'X UWWYgg'h\Y'; 9BF9'@6F5FMcb'h\Y'AccXY Wci fgY'g]hY'Zcf'Yl Uad'Yg'cZZYUhi fY'Ufh]WY'[YbfYg VYZcfY'W\ccg]b['h\Y'[YbfY'h\Ymk]``'UhhYadh'Zcf 5gg][baYbh'CbY'fcd]b]cbz'UbU`mg]gz'Yld`U]bYfz fYj]Yk'cf']aaYfg]cbŁ'''D`YUgY'U`gc'j]Yk'h\Y Zc``ck]b['j]XYcg. K\Uh]ZkY'\Y`dYX'fYZ[YYg'hc'\Y`d h\YagY'j Yg3''5`Yl UbXYf'6Yhgz'%'''%%''%)



	GI 6A ≠I'J =5 'H<9'A CC8@9'DCFH: C@€ '&'@-B? I B89F 'H<9'5GG9GGA 9BH'H56
	6F +B; 'HC'7 @5GG.'''H\fYY'fl E'Wcd]Yg'cZmci f 5gg][ba Ybh'CbY'ghcfm'dfcdcgU'
	F 958% '69'F 958M'HC '8 - G71 GG. ''F]\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
@YWhifY	Cd]b]cb'j Yfgi g'UbU`mg]gžfYj]Yk g'j Yfgi g Yl d`U]bYfg
	±b.'h\]g.'WUgg.'k Y.'YI d`cfY.'h\Y.'fYgYUfW\'.'UbX.'k f]h]b[df]bW]d`Yg.'Wca a cb`m'Udd`]YX.']b.'UbU`mh]WU`ž YI d`UbUhcfm'UbX.'Yj U`i Uh]j Y.'ZYUhi fY.'k f]h]b["' <ck [fYUh'ZYUhi fY.'k f]hYfg.'"ZFUa Y"'h\Y]f`ghcf]Yg"</ck
Hi hcf]U`	H\Y'5fhcZh\Y'": c``ck 'l d"
	K Y``]j Y`]b`ghcfmf]WX`h]a Yg"`>i ghicdYb`mci f`Hk]hhYf ZYYX`UbX`]higYYa g`h\Uh'U``h\Y`ghcf]Yg`\Uj Y`U`fYUXm

		cZVY]b['di V`]g\YX3	
		+"````K]``mci hY``h\Y`ghcfm]b`%ghdYfgcb`cf'' fX dYfgcb3`9Id`U]b`h\Y`fYUgcbg`Zcf`mcif`W\c]WY"	
< YY_'(.', 'A UfWX'!'%& A UfWX	<ca cf_<="" td="" yk=""><td>6F=B; 'HC'7@5GG."H\fYY'flEIF@jcZfYWbhmdiV]g\YX'ghcf]Yg'h\Uha UhW\'h\Y'[YbfY'cZmcifdfcdcgYX'ZYUhifY'Ufh]WYž'UbX'h\fYY'flEch\YfIF@gcZfYWbhmdiV']g\YX'bYkg'ghcf]Yg'cb'h\Y'hcd]Wmcik]g\'hc'kf]hY'UVcih'</td><td></td></ca>	6F=B; 'HC'7@5GG."H\fYY'flEIF@jcZfYWbhmdiV]g\YX'ghcf]Yg'h\Uha UhW\'h\Y'[YbfY'cZmcifdfcdcgYX'ZYUhifY'Ufh]WYž'UbX'h\fYY'flEch\YfIF@gcZfYWbhmdiV']g\YX'bYkg'ghcf]Yg'cb'h\Y'hcd]Wmcik]g\'hc'kf]hY'UVcih'	
		F 9587 '69'F 958MHC '8 €71 GG.'	
		F]W_Yhgcb'/'; fU\Ua'f&\$%+Ł'7 < 5DH9	‰



	@Ug_Ugž">YUbbY"A Uf]Y"f&\$%&Ł""; i bg"'F"I g"""; E ž
I	

; FCID'57H∌ ≠HM
HYghmci f'g_]``g'Ug'Ub'YX]hcf'UbX'dfccZfYUXYf'Vm fi bb]b['mci f'YmY'cj Yf'XfUZhg'cZdfYj]ci g'ghi XYbhgfi ghcf]Yg"'H\Yb'fYUX'U'ZY```ck 'ghi XYbhg'ghcfmXfUZh'UbX g\UfY'mci f'gi [[YghYX']a dfcj Ya Ybhg'k]h\'ZY``ck ghi XYbhg'UbX'h\Y'WUgg"

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

5```ghi XYbhg`a i ghidi f\\X\UgY`h\Y`Zc``ck]b[`hYI h\/cc_'k\]\\X`]g`gYhfYUX]b[`]b`a cghik YY_g`cZh\]g`\\circle fgY.

@Ua V`Yž`G"`f&\$%+Ł`BYk g`5g`±i< UddYbg'!`5b`±blfcXi Wfcb`hc`>ci fbU`]ga ž`C I Dž'A Y`Vci fbY"

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FYWca a YbXYX FYgci fWyg

I BGK '@-6F5FMG9FJ=79G

 $5\colon 2009 \colon 2009 \c$

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9B; @=G< @5B; I 5; 9 GI DDCFH

9bfc`a Ybh']b'l BGK 'g'D9@9'DFC; F5A 'UbX'fUb[Y'cZch\Yf'ZfYY'9b[`]g\'Wcbj YfgUh]cb'WUggYg']g $<\ \neq\ <$ @MF97CAA9B898'Zcf'ghi XYbhg'Zfca '9b[`]g\'Ug'U'GYWcbX'@Ub[i U[Y'VUW_[fci bXg'']]

.

7 ci fgY 9j U`i Uh]cb UbX 8 Yj Y`cda Ybh

Gh XYbhZYYVUW_`]g`]bj]hYX'j]U'h\Y'a m91 dYf]YbWl'gi fj Ym]b'h\Y'gYWcbX`\U`ZcZhYfa "': YYXVUW_`Zca dfYj]ci g'mYUfg`\Ug'Wcbhf]Vi hYX'hc'W\Ub[Yg`]b'W ff]Wl'i a 'UbX'UWfj]hJYgž'UbX']g'Ub']a dcfhUbhdUfhcZh\YXYj Y`cda YbhUbX']a dfcj Ya YbhcZWci fgYg"

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Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

-Zmci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\'Hi fb]h]bž'd`YUgY hY`Yd\cbY'9I hYfbU``Gi ddcfhcb'-',)'''' %cf'Ya U]`'h\Ya 'cb'YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'ghi X]Yg'UhI BGK "'\\hd.#gi VYWn[i]XYg"`]VfUfm'i bgk "YXi "Ui #\`]gY#UVci hY`]gY

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: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['lc.