

# **MDIA2013**

AU\_]b['J]fhiU`FYU`]mi8cWaYbhUf]Yg

HYfa 'C bY''##'&\$&%

## 7ci fgY'Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
? Yff]Y`8 Uj ]Yg	? "XUj ]Yg4 i bgk "YXi "Ui	H67		G5A c <i>Z</i> JW.*%& -',)'(,)*

#### GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&L'-',) '(,) \*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. 'k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$ 

## 7ci fgY 8YHJ] g

### 7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y 7ci fgY

HcXUmig`XcW a YbhUfmightfinhY``]b[ 'Zcfa Uhg`bck ']bWi XY'' \*\$'j ]XYcgž5i [ a YbhYX'FYU`]hmfbF Ł'UbX'J ]fhi U`FYU`]hmflJ F Ł"'BYk g`cf[ Ub]gUh]cbg`UfY'fY[ i `Uf`mdfcXi W]b[ 'g\cfh'J F 'ghcf]Yg`UWfcgg`X]j YfgY'hcd]Wg`UbX'Bch : cf!DfcZ|hcf[ Ub]gUh]cbg`UfY`\UfbYgg]b[ 'h\Y'dck Yf`cZj ]fhi U``fYU`]hmUg`'Ya dUh\ma UW\]bYg''hc'dfca chY Uk UfYbYgg`cZ]ggi Ygž'XYYdYb`\_bck `YX[ Y`UbX'YbWci fU[ Y`gi ddcfh''H\Y'J F 'Zcfa 'g`dcdi `Uf]hm]g i bXYfd]bbYX'VmUWWgg]V`Y'j ]Yk ]b[ 'cb'Mci Hi VY'd`UhZcfa g`UbX'ga Ufhd\cbYg`Ug`k Y```Ug'J F 'XYX]WUhYX XYj ]WYg"

H\]g`Wci fgY`dfcj ]XYg`mci `k ]h\`h\Y`dfUWf)WU``g\_]``g`]b`J ]fhi U``F YU`]mmfJ F ŁXcW a YbHJfmdfcXi Wf]cb`h\Uh'UfY ]bWfYUg]b[ `mdUfhcZh\Y`a i `h]d`Uhzcfa 'a YX]U`Ybj ]fcba Ybh'`Mci `k ]```XYj Y`cd`\UbXg!cb`WfYUfj Y`UddfcUW\Yg hc`J F `]a a Yfg]j Y`XcW a YbhJfmghcfmhY``]b[ `Zfca `]XYUZ`gWf]dhzj ]XYc]b[ `UbX`bUffUfjcb`UbX`YX]h]b[ `hc`Z]bU` j ]XYc"'H\Y`Wci fgY'g`dfUWf)WJ``Y`Ya Ybhg`k ]```VY`gi ddcfhYX`Vmh\YcfYh]WJ``fYUX]b[ gzïX]gW gg]cb`UbX`UbU`mg]g cZ]a a Yfg]j Y``cb[!Zcfa `XcW a YbhJf]Yg`UbX`h\Y]f`Wca a i b]WJh]cb`UbX`ghcfmhY``]b[ `ghfUhY[]Yg"

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 8Ya cbglfUhY`dfUVVjVW)`ž'hYVX.b]\W\``UbX`Wca a i b]\WUhjcb`g\_]``g`]b`h\Y`dfcXi Whjcb`cZJ F`XcW`a YbhUfm' &" 5dd`mh\YcfYhj\W\``i bXYfghUbX]b[g`cZ]a a Yfg]j Y`ghcfmhY```]b[`]b`XcW`a YbhUfm'UbX`Wca a i b]\WUhjcbg hc`dfcXi Whjcb`dfUVVj\W\"
- " DfcXi WY'U'J F 'j ]XYc'Zcf'U'a YX]U'dfcXi Wfjcb'dcfhZc`]c'UbX'#cf'di V`]WW\UbbY`:gi W\'Ug'Mci Hi VY" (" 7 f]h]ei Y'UbX'g]hi UhY'h\Y'\]ghcfmUbX'h\YcfmcZ]a a Yfg]j Y'ghcfmHY``]b[ 'k ]h\]b']bX]j ]Xi U``a YX]U dfUWfJWY"

### HYUW ]b[ 'GhfUhY[ ]Yg'

H\]g`Wci fgY`dfcj ]XYg`mci `k ]h\`dfUWf]WU``g\_]``g`WYUh]b[ `J F `ghcfmhY``]b[ `Zcf`h\Y'`ci fbU`]ga `UbX Wca a i b]WUh]cb`]bXi ghf]Yg"`8i f]b[ `h\Y`Wci fgYz`mci `k ]```dfcXi WY`U`J F 'j ]XYc`h\fci [\`\UbXg!cb``UV gYgg]cbg`UbX`Z]Y`X`k cf\_"'Nci `k ]```\Uj Y`Yei ]hUVY`UWWgg`hc`Yei ]da YbhUbX`dcghldfcXi Wf]cb`YX]h]b[ ZUW]`]h]Yg"`H\Y`dfUWf]WU``k cf\_`k ]```VY`gi dd`Ya YbhYX`k ]h\`h\YcfYh]WU``X]gW gg]cb`UbX`UbU`mg]g`cZ]a a Yfg]j Y ghcfmhY``]b[ `hc`Yb\UbW'`mci f`dfcXi Wf]cb`g\_]``g"

## 5ggYgga Ybh

5```UggYgga Ybhg'a i gh'VY'gi Va ]hhYX"'5'I : '[ fUXY'a Um'VY'Uk UfXYX']Z'U```UggYgga Ybhg'UfY'bch gi Va ]hhYX"

H\Y`FcVYfhiK YVghYf``UVg`UfY`UjU]`UV`Y`&(`#+`hc`ghi XYbhg'j]U`mci f`ghi XYbhiWUfX`Zcf`YX]h]b[`cb`DfYa]YfY Dfc"'D`YUgY`Ybgi fY`mci f`WUfX`[]j Yg`mci `UWWYgg`hc`h\Y``UVg"

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hJg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
JF DfcXi Wjcb fl fci dŁ	' \$ı	\$(#\$(#&\$&%`%%\$\$`DA	%ž`&ž`'
JF DcghDfcXi Wjcb flbX]j ]Xi U't'	(\$1	&' #\$(#&\$&%'%%) - 'DA	%ž'&ž''
FYZYWjcb'DUdYf'f\bX]j ]Xi U't'	' \$ı	% #\$(#&\$&%`%%) - `DA	&ž˙(

#### 5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% JF DfcXi Whjcb fl fci dŁ

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ] g.

Ga U```[ fci d`f&!(Ł'j ]XYc]b[ `UbX`]bhYfj ]Yk g`Zcf`U`h\fYY`hc`Z|j Y`a ]bi hY`J F 'j ]XYc": YYXVUW\_`dfcj ]XYX`j ]U @A G"

#### 5XX]h]cbU`XYhJ]`g.

GYY'A ccX`Y'Zcf'a cfY']bZcfa Uh]cb"

Gi Va ]gg]cb bchYg. GYY A ccX`Y

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh& JF DcgHDfcXi Wgcb f\u00abX]j ]Xi U\u00e4

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\."H\fYY"hc"ZjjY"a]bi hYg

8YHJ]`g.

9X]h]b[ 'UbX'gci bX'dcghdfcXi Wh]cb'cZh\fYY'hc'Zjj Y'a ]bi hY'XcW a YbhUfmJ F'j ]XYc'k ]h\ 'bUffUh]cb" Gi Va ]hhYX'k ]h\ 'bUffUh]cb'gWf]dhifUbgWf]dhifUddfcI "'%\$\$\$'k cfXg\!": YYXVUW\_dfcj ]XYX'j ]U'@A G"

#### 5XX]h]cbU`'XYhU]`g.

⊋mci 'fYei ]fY 'UXj ]WY 'UVci hYX]h]b[ 'cf'ch\Yf'hYW\b]WU``Ugg]ghUbWY 'fY`Uh]b[ 'hc'mci f'dfc'YWhci hg]XY 'h\Y hi hcf]U'žd`YUgY 'gYY'h\Y 'HYW\b]WU``F Ygci fWY '7 YbhYY fHF 7 Ł"'

'\*\$"WUa YfU'\_]hg'gi dd`]YX'j ]U'h\Y'HF7"cUbg'ci hg]XY'hi hcf]U'g"'GYY'A ccX`Y'Zcf'Z fh\Yf']bZcfa Uh]cb'cb Vcffck |b["

FcVYfhK YVghYf``UVg'UfY'Uj U]`UV`Y'Zcf'YX]h]b['ci hg]XY'hi hcf]U`g"`

GdYWJU Wcbg]XYfUhJcb j ]U a m bgk ]g fYei ]fYX Zcf Yl hYbg]cbg"

.

Gi Va ]gg]cb bchYg. GWf]dhbc `cb[ Yf fYei ]fYX"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh' . FYZYWncb DUdYf flbX]j ]Xi U'Ł

GhUfhiXUhY. Bchi5dd`]WUV`Y

@Yb[h\. '%) \$\$ k cfXg

8YHJ]`g.

%) \$\$!k cfX fYZYWqcb": YYXVUW\_dfci ]XYX i ]U @A G"

#### 5XX]h]cbU`XYhU]`g.

<Ufj UfX F YZYfYbW|b[ 'cf'5D5 'fYei ]fYX"</pre>

GYY'A ccX`Y'Zcf'Z fh\Yf'UggYgga Ybh]bZcfa Uh]cb"

#i]gfYWca a YbXYX mci kf]hY mci ffYZYWf]cbdf]cfhch \YXi YXUhY"

Gi Va ]gg]cb bchYg. gYY A ccX`Y

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$ 

### 5 http://bxubww.F.Yei ]fYa Ybhg

 $\label{eq:continuity} D`YUgY`bchY`h\Uh`YWh fY`fYWcfX]b[g`UfY`bchUj U]`UV`Y`Zcf`h\]g`Wci fgY"`Ghi XYbhg`UfY`ghfcb[`miYbWci fU[YX`hc UhhYbX`U```WUggYg`UbX`WcbhUWhh\Y`7ci fgY`5i h\cf]mnhc`a U_Y`U`hYfbUh]j Y`UffUb[Ya Ybhg`Zcf`WUggYg a ]ggYX"$ 

## 7 ci fgY GWX YXi `Y

#### J]Yk Wugg'ha Yhuv'y

#### H]a YHJV`Y

8UHY	HmdY	7 cbhYbh
K YY_'%'%) ': YVfi Ufm!'%- : YVfi Ufm	Gh X]c	BHFC8I7H€B. K\miHY```]a a Yfg]j Y`ghcf]Yg3 B'ge] Wk VW(`[f[a`el[`fdaVgUf[a` fa SeeWee_ W f efack fab[UlS`V[`fdaVgUf[a` fa;`efS %(" A`WJ US_ WbSež
		@afW kag`WW Se_SofbZa`WXadfZ[e UagceW/adSf WSefa`WbWdea`[ kagdYdagbfifa Ua``WUffa fZW US_WdSž
		FYUX]b[g. 'gYY'AccX`Y"'H\Y'hYIhVcc_']g'fYei]fYX fYUX]b["
		HYI NVCC `6cgk cfth. `A "ž/ `GUfU\ž'@'`f&\$% t"` 5c6xf[`Y Efac[We XadH[dfgS^DW\$/ffk"'CIcb`#BYk Mcf `FcihYX[Y"
K_YY_"&."&&": YVfi Ufm!"&*	Gh X]c	JF GHCFMH9@@B; GHM@9G
: YVfi Ufm		=a a Yfg]j Y'bUffUh]j Yg'UbX'bYk g'UbX'ch\Yf ghcfmhY``]b['ghm`Yg' FYZYf'hc'A ccX`Y'Zcf'FYUX]b["'HYIHVcc_'fYei]fYX fgYY'UVcj YŁ"
		GhcfmUbX'Zj`a 'k cf_g\cd"
K YY_" ."%"A UfVX"!") A UfVX	Gh X]c	JF 'ghcfmh/``]b[ 'ghm`Yg'dUfh'& 'Bch'Zcf'DfcZ]h ghcfmh/``]b[ 'UbX'h\Y'fi'a dUh\ma UVX]bYfi
		FYZYf'hc'AccX`Y'Zcf'fYUX]b[g"
		; fci d'Zj`a 'k cf_g\cd'UZrYf'UVcj Y. '7 cc[ YY'VYUW\ fk YUh\Yf'dYfa ]hjb[ Ł"'A YYh]b'WUgg'Zjfgh'
K YY_'(.', 'A UfVX'!'‱ A UfVX	Gh X]c	5i X]c ∈a a Yfg]j Y GhcfmHY``]b[
		Gci bX'UbX'bUffUhjcb'k cf_g\cd'
i	I	ı

		FYZYf'hc'AccX`Y'Zcf'fYUX]b[g"
K YY_") ."%) "A UfVX"! "%- A UfVX	Gh X]c	; fci d'K cf_g\cd'Zj`a ]b[ '
PA OTVM		A YYh]b`WUgg`Zcf`Vf]YZ]b[`i b`Ygg`ch\Yfk]gY UffUb[YX"
		D`YUgY`\Uj Y`U`a Ug_`Zcf`di V`]WhfUbgdcfhifffUa Łhc 7]fW``Uf`E i Um'
K YY_'*.'&&'A UfWX'!'&* A UfWX	F YUX]b[	D`YUgY'i gY'h\]g'k YY_'hc'k cf_'cb'mci f'[fci d dfc^YWh'UbX'U`gc'd`Ub'mci f']bX]j]Xi U``a U^cf dfc^YWh'
K YY_'+.'&- 'A UfVX'!'& 5df]`	Gh X]c	
		9X]r]b[ '=a a Yfg]j Y'GhcfmtY``]b[
		9X]l/jb[ 'k cf_g\cd'
		; fci dʻUggYgga YbhYl YfW]gYʻXi Yʻh\]gʻk YY_"'
K YY_', .') '5df]`'!'- '5df]`	Gh X]c	9h\]WgʻUbXʻ∍a a Yfg]j YʻGhcfmhY``]b[
		FYZYfihciAccX`YiZcfifYUX]b[g"
		A U⁄cf glcfmk cf_g\cd ha Y
K_YY_''%%'5df]`'!'%*	Gh X]c	-a a Yfg]j Y'J F 'ghcfm'hY``]b['
[5df]`		C W `i g`F ]ZhUbX`J F `XcW a YbhUfmf7 cj ]X!% fYghf]Wf]cbg`XYdYbXYbhŁ`
		FYZYf'hc'a ccX`Y'Zcf'fYUX]b["
K YY_"%\$."%-"5df]`"!"&' 5df]`	Gh X]c	9X]h]b[ a U^cf'ghcfmk cf_g\cd'UbX'YggUm ffYZYWh]cbŁk cf_g\cd'h]a Y"
		Bc fYUX]b[ g"

## Fygci fWyg

## DfYgWf]VYX F Ygci fWYg

 $6 \text{cgk cfh} \tilde{z} \text{A "} \tilde{z} \text{UbX'GUfU} \tilde{z} \text{@"'f\&$\% -L"'} 5 \text{dSXf[' Y E fadWe XadH [dfgS^D WS fk z'B M \#C I cb. 'F ci hYX[ Yz'Uj U]'UV'Y j ]U'I B G K "Vcc_g \ cd'h\ Uh'U'gc'c ZYfg'Y6cc_'di fW\ UgYg'''$ 

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/