

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

A YX]Užī7 i `hi fY`UbX`9j YfmXUm@]ZY`cZZYfg`Ub`UWWgg]V`Y`[fci bX]b[`]b`h\Y`HfUX]h]cbgž'dYfgdYWhj Yg`UbX WcbW'dhg`cZa YX]U`ghi X]Yg"`GhUffi]b[`k]h\Y`XJUa]`]Uf`UbX`h\Y`Yj YfmXUmž'h\]g`Wci fgY`]bHcXi WYg`mci `hc`h\Y VfYUXh\`cZWcbhYa dcfUfma YX]U`ghi X]Yg`Zca `hY`Yj]g]cb`UbX`h\Y`HfUX]h]cb`cZa Ugg`a YX]U`ghi X]Ygž'hc hY`Yd\cbmUbX`h\Y`ghi XmcZbYhk cf_YX`a YX]U`UbX`Wca a i b]WUh]cb`hYW\bc`c[]Yg"`K]h\`Ub`YmY`cb`h\Y`k Um h\Uh'hY`Yj]g]cb`UbX`hY`Yd\cbm\Uj Y`UXUdhYX`hc`W\Ub[]b[`fYU`]h]Yg`cj Yf`h\Y`XYWUXYgž'h\]g`Wci fgY`Yl d`U]bg \ck `a YX]U`UbX`Wca a i b]WUh]cb`hYW\bc`c[]Yg`\Uj Y`HfUbgZcfa YX`h\Y`f\mh\a g`cZYj YfmXUm`]ZYž'h\Y cf[Ub]gUh]cb`cZXca Ygh]WgdUVWž'h\Y`Vci bXUf]Yg`VYhk YYb`df]j UhY`UbX`di V`]Wž'UbX`ci f`gYbgY`cZ]bj c`j Ya Ybhk]h\`bUh]cbU``UbX`di V`]WWc``YWhj]h]Yg"`\d\cbmg\UdY`h\Y`Yl dYf]YbW'`cZha YžX]ghUbW'ž a YX]Uh]cb`VmYl d`cf]b[`\ck `Vch\`hY`Yj]g]cb`UbX`hY`Yd\cbmg\UdY`h\Y`Yl dYf]YbW'`cZha YžX]ghUbW'ž

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±b!7`Ugg'Ei]n'&'	%\$I	7 ca d`YhY`Xi f]b[`K YY_`,	% <u>*</u> ''
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 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$

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 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b "gYhh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hYX"h\fci [\ 'Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b g]a]`Uf]hmfYdcfhg" \\ \end{tabular}$

Hi fb]h]b`gYh]b[.`H\]g`Ugg][ba Ybh]g`gi Va]hYX`h\fci [\`Hi fb]h]b`UbX`gh XYbhg`Xc`bchgYY`Hi fb]h]b g]a]`Uf]hmfYdcfhg"

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K YY_'%'%)': YVfi Ufm!'%- : YVfi Ufm	@YWnifY	A YX]U'F]hi U`g'UbX'9j YfmXUm@]ZY
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		<pre>%" 5WUXYa]WfYUX]b['g_]``g'UbX`ghfUhY[]Yg &" K \UhXc'k Y'a YUb'VmfYj YfmXUm`]ZYfB ' " 8]ZZYfYbhUddcfUWXYg'hc'h\Y'gh XmcZa YX]U" (" < ck ']g'h\Y'gh XmcZa YX]U'X]ZZYfYbh'Zfca ch\Yf'X]gWjd`]bYg'gi WX'Ug'dgmWXc`c[m²</pre>
		Wf]a]bc`c[mcf`gcWjc`c[m8) "K\Uh']g`U`f]h U`3`K\Uh']h U`g`gi ffci bX`ci h i gY`cZh\Y`a YX]U`UbX`k\UhWUb`kY``YUfb Zfca 'h\Ya 3
K YY_'&.'&&': YVfi Ufm!'&* : YVfi Ufm	@YWn fY	A YX]U'UbX'Df]j UhY#Di V`]W@]ZY
		5a cb['ch\Yf'h\]b[gž\]g'k YY_fg'hcd]W'cc_g'Uhh\Y \]ghcf]WU''fY`Uh]cbg\]d'VYhk YYb'f\ca YgfiUbX'a YX]U hYW\bc`c[]Yg"'K Y'k]``'Uf[i Y'h\Uha YX]U']g'VYgh i bXYfghccX'Zfca 'h\Y'dYfgdYWf]j Y'cZh\Y'df]j UhY W'`hi fYg'cZi gYžVmk\]W\'kY']bWcfdcfUhY'a YX]U']bhc ci f'XU]`mfci h]bYg'UbX'YI dYWfUh]cbg"'
	Hi hcf]U`	→ 'h\]g'k YY_fig'h hcf]U`g'k Y'k]``fYZYWicb'h\Y fY`Uh]cbg\]d'VYhk YYb'mci f'fica YfiUbX'h\Y'a YX]U'cZ h\Y'fdii V`]WfiE i Ygh]cbg'Zcf'h\]g'k YY_fig'h hcf]U``UfY. K\Uh\Wb'h\Y'WcbWYdhcZXca Ygh]WUh]cb'hY``i g'UVci hh\Y'a YX]U3' <ck '\uj=""]b'hyfa=""]b[=""]n']b'k="" bxuf]yg'vyhk="" dwca="" e="" fig'wugg"<="" g'cza="" gg'h\y'i="" i="" td="" ubx'df]j="" udd]b['h\y'vci="" uhy3'k="" v`]w="" y="" y'h\y'a="" y'k]``'u`gc'x]gw="" yfh]a="" yx]u'w\ub[yx'cj="" yy_''="" yyb'di=""></ck>
K YY_'' .'%'A UfW\'!') A UfW\	@YWi fY	A YX]Uh]cb. H\Ya YUb]b[cZh\YA YX]U
		H\]g'k YY_'k Y'k]``YI d`cfY'h\Y'WcbW'dhfA YX]Uh]cbfi dUfh]W'`Uf'm'Vm`cc_]b['Uh'h\Y'fY`Uh]cbg\]d'VYhk YYb a YX]U'UbX'ci f'YI dYf]YbW''cZh]a Y'UbX'gdUW'"

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	Hi hcf]U`	H\Y'a U]b WcbWdhg'YI d`cfYX'h\]g'k YY_']b h hcf]U`g]bWi XY/h\Y'fc`Y'cZgcW]U`'a YX]U']b Yj YfmXUm`]ZYž k\Uh]g'a cV]`]hmž']b k\Uhk UmXcYg'a cV]`Y'a YX]U ZUW]HUHY'a cV]`]hm"K Y'k]``U`gc'Wf]hJWU``m'YI Ua]bY'h\Y fc`Y'cZU`[cf]h\a g'UbX']b'a YX]U'hcXUm"
		K_Y`U`gc`Wca_d`YhY`h\Y`gYWcbX`ei_]n`h\]g`k_YY_"`
K_YY_''\&'5df]`'!'* 5df]`	@YWhi fY	A YX]UžGY`ZUbXʻ≼Ybh]hm
		Cif``Ugh`]jY``YWifY``cc_g`Uhih\Y`fY`Uh]cbg\]d VYfk YYb`]XYbf]fmUbX`a YX]U"`K Y`k]```Ug_`k\Uhifc`Yž]ZUbmžfh\Y`a YX]U`d`Um]b`dfcXi W]b[`UbX dYfdYh Uh]b[`fh\Y`WUgg]Z]WbcfmfY[a]Yg`Vmk\]W\`kY WUhY[cf]gY`UbX`i bXYfgHUbX`ci fgY`j Yg`UbX`YUW\ ch\Yf" .
	Hi hcf]U`	Hi hcf]U`gʻh\]gʻk YY_ʻk]```X]gW ggʻh\Yʻfc`YʻcZa YX]Uʻ]b dfcXi Wjb[ʻUbX`YbUWjb[ʻmci fʻ]XYbhjhm''5fY`]XYbhjhjYg X]gW fg]j Y`mdfcXi WYX3'5fY`h\Ym]bX]j]Xi U`]gYX`cf Wc``YWjj]g]b[3
K_YY_`%\$.`%-`5df]``!`&' 5df]`	@YWn fY	FYj]Yk @YWn fY'! =b'h\]g``YWn fY'k Y'k]```fYj]Yk 'h\Y hcd]Wg`Wcj YfYX']b'h\Y'Wci fgY'UbX'Ubgk Yf'mci f ei Ygh]cbg']b'fY`Uh]cb'hc'h\Y':]bU`'9ggUmXi Y'bYI h k YY_"'
	Hi kcf]U`	H\]g`kYY_fg`@YWn fY`k]``fYj]Yk`H\Y`WcifgY`VUgYX cb`mcif`ZYYXVUW_`UbX`eiYgH]cbg`UbX`cZZYf`HJf[YHYX Ugg]gHJbWY`k]H\`H\Y`:]bU``9ggUmUggYggaYbH'

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FYWca a YbXYX FYgci fWYg

9@G9'!'@]VfUfm'UbX'Ghi XmG_]``g'8Yj Y`cda Ybh'5@ii bXYf[fUXi UhY'ghi XYbhg']b'h\Y]f']fghgYa YghYf'bYYX hc'Wa d'YhY'h\Y'`]VfUfmfg'9@G9'ghi Xmg_]``g'hfU]b]b['a cXi `Y'VYZcfY'h\Y'YbX'cZK YY_') "'GYY'h\Y''9`]gY''hUV cb'h\Y'`]VfUfm\ca YdU[Y.`\hd.#gi VYWf[i]XYg"`]VfUfm'i bgk "YXi "Ui #\`]gY'Zcf'] fh\Yf']bZcfa Uh]cb"

: i fh\Yf`]bZcfa Uh]cb`UVci hih\Y`@]VfUfmig`Wc``YWf]cb`UbX`gYfj]WYg`WJb`VY`Zci bX`\YfY`! \hd.#|bZc"`]VfUfmii bgk "YXi "Ui #\ YV#gYfj]WYg#gYfj]WYg"\ha`

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Gh XYbhZYXVUW_`]g`hJ_Yb`gYf]ci g`mžUbX`Wcbh]bi U``]a dfcj Ya Ybhg`UfY`a UXY`hc`h\Y`Wci fgY`VUgYX`]b`dUfh cb`gi W\`ZYYXVUW_"DfYj]ci g`gh XYbhZYXXVUW_`UVci hh\Y`Wci fgY`fYUX]b[g`UbX`UggYgga YbhHJg_g`\Uj Y VYYb`UXXfYggYX`]b`h\]g`j Yfg]cb`cZh\Y`Wci fgY"`Mci `WJb`fYUX`U`XYgWf]dh]cb`cZW\Ub[Yg`k Y`a UXY`]b fYqdcbqY`hc`dUghqh XYbhZYXVUW`]b`h\Y`A ccX`Y"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb['tc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\ 'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
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IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

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