

ARTS3049

@]hYfUfm5b]a U`gž'A cbghYfg'UbX'A UVX]bYg

HYfa 'C bY''# '&\$&\$

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

 $K \] XYgdfYUX'] b'fh Y' \ i \ a \ Ub] fi Yg'Uhfh Y' a \ ca \ Ybh'] g'U'fYhh] b \] b \ ['cZk \ Uh'] ha \ YUbg'hc'VY' \ i \ a \ Ub''' + b'h \] g'Wc' fgY mc' i k]'```cc_'Uh' \ ck '`] hYfUh' fY'UbX'\ I a 'i gY'h Y'bcb\ i a Ub''gi V'YWfj] mcZUb] a U'gž'a cbghYfg'UbX' a UW] bYg hc'g \ UdY'ci f'i bXYfgHUbX] b \ ['cZh Y'\ i a Ub'''6mYI Ua] b] b \ ['hh Y'W Ub['] b \ ['dfYgYbHUf] cb'cZUb] a U'gž'a cbghYfg \ ['hh Y'W Ub''] b \ ['hh Y Ub'' Ub''] b$

5ggYgga Ybh

Dfc YWiA i gY fa i gY"\i "YXi E'UbX 'ghcf'flghcf"cf[E'UfY YI WY``YbhfYgYUfW\ XUHUVUgYg 'Zcf'mci f UggYgga Ybhg"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
@cb['9ggUm'	* \$1	\$) #\$) #&\$&\$`\$) .\$\$`DA	%ž&Ž
F YgYUfWX '91 YfWjgY '	(\$1	%\$#\$' #&\$&\$`\$).\$\$`DA	%ž&Ž

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh'% '@cb['9ggUm

GHJfh'XUhY.

@Yb[h\.'&) \$\$ k cfXg

8 YHJ]`g. `&) \$\$`k cfX`YggUm'': YYXVUW_'dfcj]XYX`j]U`Hi fb]h]b"

5XX]h]cbU`'XYhU]`g.

H\]g`]g`U`&) \$\$!k cfX`YggUmžXi Y`cb`Hi Yg`A Um) `fk YY_'%&L"Bc`ei Ygh]cbg`k]```VY`gYhZcf`h\]g`Ugg][ba Ybh' Mci `UfY`YI dYWYX`hc`XYj Y`cd`mci f`ck b`ei Ygh]cb`]b`Wcbgi `hUh]cb`k]h\`mci f`h hcf"'\domai f`YggUmž'mci `a i gh UbU`mgY`]b`XYhU]``h\ fYY`cZh\Y`Wci fgY'g`gYhhYI hgžVi himci `a UmbchX]gW gg`? UZ_U'g`"5 F Ydcfhhc`Ub 5\WUXYa m''`Ug`cbY`cZh\YgY`h\fYY''\Mci `UfY`YbWci fU[YX`hc`X]gW gg`h\Y`hYI hcb`k \]\W`mci `dfYgYbhYX`]b`h\Y gYa]bUfg"

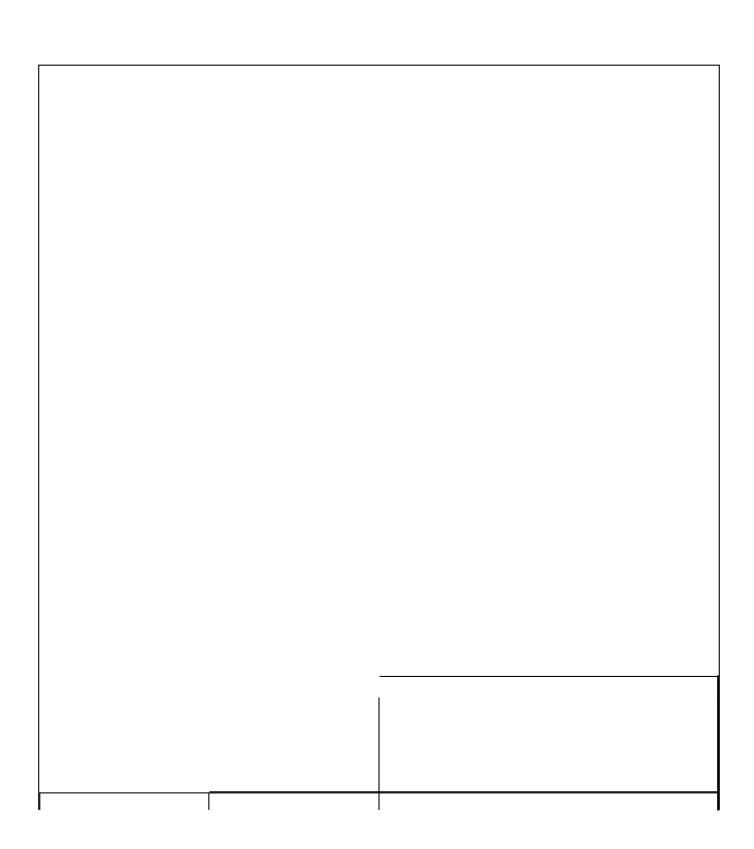
 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$

5ggYgga Ybh'& FYgYUfW '9I YfW]gY

GhUfh'XUhY.

@Yb[h\. *\\$\$\$ k cfXg

8YHJ]`g. '5 '%\$\$\$!k cfX`YI YfW]gY`dfYdUf]b['ghi XYbhg`Zcf`h\Y``cb['YggUm'`Ghi XYbhg`k]```VY`Ug_YX`hc`fYgYUfW\ gYWcbXUfmgci fWYg`UbX`]XYbh]ZmU`ei Ygh]cb`Ya Yf[]b['Zfca 'h\cgY'gci fWYg": YYXVUW_dfcj]XYX`j]U`Hi fb]h]b" $\label{eq:condition} $$hY'\in a$ Ub']b'\subseteq g\subseteq fh'gh''5'FYdcfh'hc'Ub'5WUXYa m''3$$



K YY_', . '* '5df]`'! '%\$ '5df]`	@YWhifY	FZW? WfS_acbZae[e	
	GYa]bUf	FZW? WfS_adbZae[e	
K_YY_""%"	@YWhifY	@a WUfgdWfZ[ei WW] VgWfa 7SefWd? a`VSk	
5df]`	GYa]bUf	@a eW_ [`SdfZ[e i WW] VgWfa 7SefWd? a`VSk	
K_YY_"%\$."&\$"5df]"!"&(GYa]bUf	? Wfcaba 1je	
5df]`	GYa]bUf	? Wfdaba1e	
K_YY_`%%`&+`5df]``!`&,	@YWhifY	6a3`Vda[Ve6dMS_aX7MUfd[UEZWWb1	
5df]`	GYa]bUf	6a3`Vda[Ve6dMS_aX7MUfd[UEZWWb1	
		fgi dd`Ya YbfUfmfYUX]b[Ł'=`U\'F YnU'Bci fVU_\g\ž "8Y\i a Ub]n]b['F cVcfg"	

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

Df]a UfmihYI hg

H\YgY`hYI hg`UfY`Uj U]`UV`Y`Zfca `h\Y`I BGK `Vcc_g\cd"`Mci `a Umi gY`ch\Yf`YX]h]cbgž`]Zmci `U`fYUXm`\Uj Y h\Ya "

>c\b'6Yf[Yfž'3Tagf>aa][Yfb'ccagVifmL

fBcfhcb:7>f]h]Wbh:19hX]f]kh]Zhž:9g^f[hWdeFdShWefBcfhcb:7f]h]WU:19X]h]cbŁ

A UfmiG\Y``Ymž8dS`] W efW[`fBcfhcb'7f]h]WU``9X]h]cbt"'K Yfi`VY`ig]b['h\Y'% % 'YX]h]cb'cZh\Y'hYIh'

<"'; "'K Y``gž'FZW;e'S` V aX6 aUfad? ad $\sqrt{8}$ g fbfcUXj]Yk '9X]h]cbŁ

: fUbn'? UZ_Už'? WfS_ acbZae[e S` V A fZWdE faqWe fDYb[i]b'9X]h]cbŁ

=`U\`FYnU`BcifVU_\g\ž'\8Y\ia Ub]n]b[`FcVchg""`DaTaf8gfgdWef7Ua Vf]X[Yž'A UggUW\igYhhg.`A \pm I'DfYggž&\$% \pm '(-!*("'

FYWca a YbXYX FYgci fWYg

<u>: i fh\Yf^fYWca a YbXYX^fYUX]b[</u>

8]Ua cbXž7cfU""H\Y'8]ZJW `hmcZFYU`]hmUbX'h\Y'8]ZJW `hmcZD\]`cgcd\m**z**"BSdf[S^3`ei Wde, 3 <agd S^aX >[fWdSfgdWS`V fZW: [efack aX;VWSe %"&'fbi bY'&\$\$' L'% &- "'5j U]`UV`Y'j]U'Dfc'YWfiA i gY'Uh\hd.##a i gY"\i "YXi #ci fbU`g#dUfhU`SUbgk Yfg# \$\$\##\"&"X]Ua cbX"\ha`

; i bb]b[ž'Hca "'FZW8[$\underline{\ }$ e aX8qffl >S`Y, 3 ^WYaq[We aXH[e[a`S`V? aVWd [fk fl@cbXcb.'6f]h]g\':]`a ' $\underline{\ }$ bgh]hi hYž &\$\$\!''G'+-\"('\$&' | #\%

 $Bcff]gžAUf[ch'4WSefe aXfZW? aVWd:_SY[Sf[a`,6Sd][+@[WfleUZW=SX]Sh7defh`>SidWUW'fbUh]acfY. H\Y'>c\bg'<cd_]bg'lb]]Yfg]hmDfYggž%-,) \E'', $-"-'', (#%$

BcifVU_\g\ž=`U\`FYnU"`DaTaf8gfgdMef7UaVf]X[YžAUggUWXigYhg.`A\H`DfYggž&\$%\E''

D\]'a i gžF cVYfhA ";' fa fZWG`] 'ai `, FZW7 ha 'gf[a` aXEU[W UW8[Uf[a` Xta_ 8c6` U[e 9 aVi [fa : ž 9 ž I W'e f6Yf_Y'Ym'I b]j Yfg]hmcZ7 U`]Zcfb]U`DfYggž% +\$Ł"'G', \$-"' , +*#

K Y]'ž'? Uf]"'FZ[`][`Y 3`[_S'e, | Zk 3`[_S^EfgV[We @ai 3'fBYk 'Mcf_.'7c'iaV]U'lb]j Yfg]miDfYggž&\$\\L'5j U]'UV'Y'cb']bY'j]U'h\Y'l BGK '`[VfUfm"

K UfbYfž'A Uf]bU"'8S`fSef[U?WfS_acbZaeWeł A fZWdI adVe, I Ske aXFWY Y fZWEWXfCIZcfX. 'CIZcfX I b]j Yfq]lmDfYgqž'&\$\$+Ł"

K c`ZYž7Ufm"I ZSf;e BaefZg_S`[e_1 fA]bbYUdc`]g. I b]j Yfg]mcZA]bbYgctU'DfYggž'&\$%\$t"'%(-#(()

7 ci fqY '9i U'i Uhcb 'UbX '8 Yi Y`cda Ybh

5 ZhYf ZYYXVUW_ UVci h`UghmYUffig Wci fgYž =\Uj Y`a cj YX`7 {I STWfZ 5 aefW*a Zfca 'k YY_'% hc'k YY_'(']b h\Y WUgg gW\YXi `Y`gc h\Uhk Y`WUb`gh Xm]hg fY`Uh]cbg\]d hc '9 g ^{h\Whe F cShWe" Gh XYbhg k]```VY`YbWci fU[YX`hc Z\``ci hcb`]bY`qi fj YmZcfa g`Uhh\Y`YbX`cZh\Y`Wci fqY`gc'h\Uhi]a dfcj Ya Ybhg a UmVY`a UXY"

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

5 WUXYa]W±bZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/`UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W]Yg/
- YI Ua]bUh]cb dfcWXi fYg/